Multi-market Simultaneous Search: Theory and Application*

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Abstract

We model multi-market simultaneous search and based on it we propose a novel theory of how a temporary financial crisis can have long-lasting negative effects on the labor market and the economy, despite the fact that workers search permanently harder after the crisis. We assume two labor markets and workers can choose whether to only search in their local market or to simultaneously search in the other market. Multi-market simultaneous search imposes negative externality to other workers, because these applicants sometimes reject offers if receiving offers from both markets, which lowers firms' incentive to create vacancies. Such strategic complementarity of workers' search behaviors makes possible multiple equilibria. Large enough financial shocks that increase the entry cost of firms, even temporarily, can cause the labor market to permanently switch from the Low-Intensity-Equilibrium to the High-Intensity-Equilibrium, where workers search in both markets. Due to the negative externality, the job finding rate can be smaller and then the recovery of the labor market is slower, even though every unemployed worker searches harder. We calibrate the model to match the features of the U.S. economy, and show that the calibrated model features multiple equilibria. We further show that in response to a transitory financial shock, the model is able to generate a much larger, self-fulfilling and persistent changes in the labor market, which matches well the post-recession high unemployment rate of the U.S. labor market between 2009 and 2011, aka the jobless recovery.

Keywords: Search frictions, Unemployment, Jobless recovery, Multiple equilibria, Labor mobility.

JEL codes: E24, J64, J61.

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1 Introduction

A striking feature of the Great Recession is how a large but temporary financial crisis triggers a severe economic crisis followed by a prolonged recovery of the labor market afterward. The Federal Reserve Bank and the US government has taken unprecedented measures, such as Quantitative Easing, to revive the economy. Despite of these efforts, the unemployment rate was above 8% for 43 consecutive months and six years after GDP growth first turned positive, it was still above 5%. Many even call it the "jobless recovery". This raises some curious questions: was this recession different because it was caused by financial shocks? Is there a special link between financial shocks and the labor market? The economic hardship is also accompanied by changes of search behavior by workers. The media reported record number of job applications sent by each unemployed worker. Figure 1 shows that the time spent on job searching by the unemployed not only increased before and during the crisis but interestingly also appeared to be permanently higher afterward, according to the American Time Use Survey. Many research also find job search intensity of the unemployed in the U.S. is countercyclical or positively correlated with the local unemployment rate (e.g., Shimer, 2004; Mukoyama et al., 2014; Aguiar et al., 2013; Faberman and Kudlyak, 2014). However, the standard labor search models of search intensity, which analyses how hard a worker should search in a given market, predict that search intensity should be procyclical.

We propose a new way to model search intensity and based on it a novel theory of how a temporary financial crisis can have long-lasting negative effects on the labor market and the economy, despite the fact that workers search permanently harder after the transitory crisis. The key of our theory is multi-market simultaneous search. In other words, we allow workers to endogenously choose how many markets to simultaneously search in for jobs. We find that if a worker increases her search intensity by simultaneously search in more markets, she naturally imposes a negative externality upon other searchers, in terms of lowered job finding rate. When these other searchers find it harder to get a job in their "local" markets, they themselves have more incentive to simultaneously search in more markets. The strategic complementarity of workers' search behaviors makes possible multiple equilibria: a low search intensity equilibrium (LIE) and a high intensity equilibrium (HIE). Based on these results, a theory of dynamic bifurcation is then proposed: large enough financial shocks that increase the entry cost of firms, even temporary, can cause the labor market to permanently switch from the LIE to the HIE. Although workers search harder, the overall job finding rate can be smaller if the negative externality is strong. In other words, the equilibrium where everybody searches harder could have lower match efficiency in the labor market. If so, the beveridge curve shifts out and the recovery of the labor market is slower than if simultaneous search is not allowed, and the economy rests on a permanently higher unemployment rate.[1]

Our theory is motivated by the following idea: if an unemployed workers wants to search harder, especially during hard times, she not only searches harder in a given market, but may also look for jobs in other markets in which she does not normally search. A market here can be a type of occupation, an industry, or a skill level. For example, some unemployed workers put on posters that read "Will Take Any Job", during the Great Recession. One can also interpret markets geographically: an unemployed worker is choosing whether to search for the local jobs only, or the jobs in other areas, as well. Multi-market simultaneous search might not be so popular during normal times, because an unemployed worker might concentrate her search effort in the local market in order to save search and potential moving cost, or because of preference over locations. But it can be particularly important during hard times. Have workers searched more broadly since the Great Recession? We use a difference-in-difference approach to estimate how the moving rate is associated with the state unemployment rate in the US. The result is shown in Table 1. We can see that the higher the unemployment rate in one state, the more likely a working-age adult migrates within the state, either within or across the county. This suggests that when local labor market deteriorates, workers indeed tend to search more in other markets.

Our framework only deviates in one regard from the standard random search and matching models as in Mortensen and Pissarides (1994) (MP thereafter): we assume two labor markets and workers from the two markets can endogenously choose whether to only search in the "local" market or in both markets. The matching process in each market is still governed by a constant-return-to-scale (CRS) matching function. In the standard MP theory, adding one more unemployed worker to a labor market does not affect the job finding prospect of all the other workers in the same market due to the CRS matching function. This is because under the CRS assumption, both firms' entry decision and the job finding rate of the unemployed depend exclusively on the vacancy-to-applicant ratio, aka the market tightness, which is determined by the firm's free entry condition. The addition of that new worker will be accommodated by an increase in the number of vacancies, leaving the market tightness unchanged. The firms are willing to do so because their profit per vacancy is unchanged.

[1] On the other hand, if the negative externality is weak, then the opposite is true. Therefore, labor market fluctuations can be amplified or dampened by the endogenous simultaneous multi-market search behaviors.

But when workers start simultaneously searching in the markets which they do not normally search, they "pollute" the original pool of applicants faced by firms in those markets. Specifically, if a worker searches in both markets simultaneously, naturally there is some probability that she finds a match in each of the market, at the same time. That means one of the two offers will be rejected. In other words, when workers search more broadly, more matches will be wasted. Lowered market tightness (the vacancy-to-applicant ratio) is then needed to compensate firms from the fact that now more of their offers will be rejected. At the mean time, lower market tightness also means lower job finding rate for other workers. This is the negative externality among workers' search behavior imposed by simultaneous search. We also show that even if the two markets are identical and moving is not costly at all, such externality is still present. Because simultaneous search is only useful when one cannot find jobs in the "local" market, the lowered market tightness in the local market hence make workers more prone to search in the other market. Strategic complementarity thus makes multiple equilibria possible.

We first demonstrate the mechanism of simultaneous search in a static setting, and then extend the analysis into a dynamic environment. Suppose we have multiple steady state equilibria and begin with the more efficient LIE. As in the standard search models, the cost of creating a vacancy can be seen as an investment to obtain the potential cash flow provided by a job match. Consider a financial crisis that increases the investment/entry cost of firms temporarily. During the financial crisis, the reduced firm entry make it harder for workers to find jobs locally. If the financial shock is severe enough, we no longer have the LIE. In other words, even if everyone else only searches in local markets, it is better off for an unemployed worker to search in both markets because the job finding rate is low in her local market. Thus the economy would switch to the HIE, in which everyone searches in both markets. Now even if the investment/entry cost resumes to normal, the labor market remains in the HIE. Because switching back to the LIE would require a large positive shock to kill the HIE or an economy-wide coordination.

To our knowledge, this is the first attempt to model multi-market simultaneous search. Usually, search intensity is treated as a technology parameter (e.g., Pissarides, 2000), which increases the match probability for a worker. However, an individual worker can also improve her matching probability by searching more broadly. Many authors study this extensive margin by assuming that workers can choose how many firms to apply for (random search: Shimer, 2004; directed search: Gautier and Moraga-González, 2005, Albrecht et al., 2006, Galenianos and Kircher, 2009, and Kircher, 2009). There are two reasons why we do not use their multi-firm framework. First, the matching function (for markets) framework is more tractable and has received extensive empirical support and

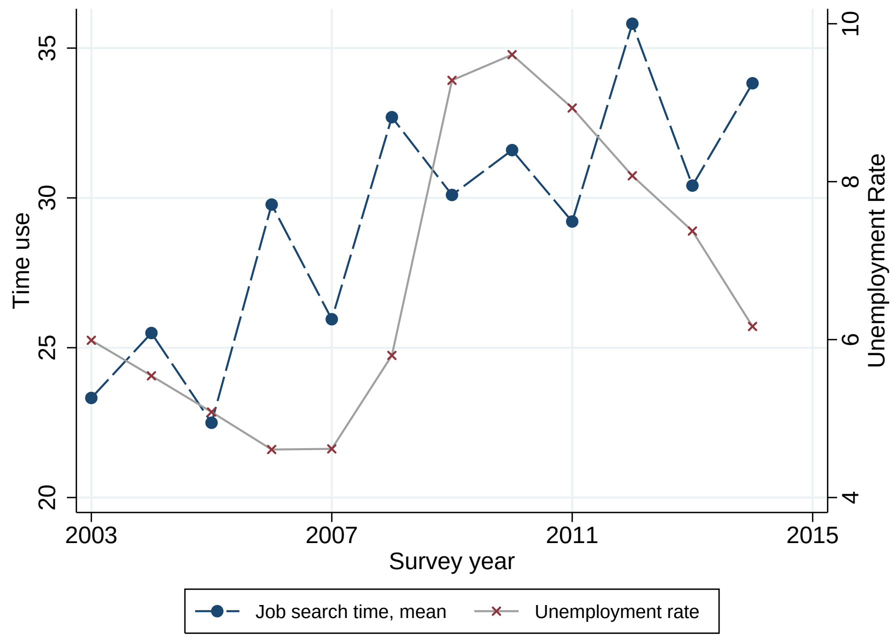

Figure 1: Time Spent on Job Search by the Unemployed

Table 1: DID: The moving rate and the unemployment rate at the state level, CPS 2008-2014.

	(1)	(2)	(3)	(4)
Moving rates	Overall	within-county	within-state across-county	inter-state
Unemployment rate	0.247**	0.145*	0.059*	0.026
	(0.069)	(0.074)	(0.028)	(0.014)
# of observations	357	357	350	357
R-squared	0.78	0.77	0.73	0.75

Notes: *** p<0.01, ** p<0.05, * p<0.1. Robust standard errors (clustered at the year level) in parentheses. The dependent variable is the moving rate. Independent variables include state fixed effects and year fixed effects.

been widely and successfully applied in macroeconomics, especially in studying business cycle fluctuations of the labor market. Second, when workers are thinking about searching in additional markets, there are at least two unique and important considerations: fixed search cost and moving cost. For instance, suppose a Boston worker wants to start searching in New York. Even if she only wishes to send one application, it is likely that she needs to do some research so as to determine which company in New York to apply for. This fixed cost of learning about a new market is usually much higher than the actual cost of sending one application. This explains the discrete nature of multi-market search. Additionally, the potential cost associated with moving is another important feature of multi-market search. If one switches occupation or industry, there usually is a transitional period when the productivity/wage of the workers is low, not to mention the obvious cost of moving if one changes geographical location.

One of our contribution is to advance a novel theory of multiple equilibria and how temporary real shocks (specifically, financial shocks) can permanently cause the switch of equilibrium. In Mortensen (1999), there is also a bifurcation system. But like many other papers, the source of multiplicity comes from increasing returns to scale in production (Benhabib and Farmer 1994, Farmer and Guo 1994, and Christiano and Harrison 1999). Multiplicity can also obtain due to increasing returns to scale in matching (Diamond 1982, Diamond and Fudenberg 1989 and Boldrin et al. 1993). In Heller (1986), Roberts (1987) and Cooper and John (1988), multiplicity obtains because of demand externalities. More recently, Kaplan and Menzio (Forthcoming) propose a theory of multiple equilibria based on shopping externalities, which is present because a firm hiring an additional worker creates a positive external effect on other firms, as a worker has more income to spend and less time to search for low prices when he is employed than when he is unemployed. Our theory is different from the above studies because here the strategic complementarity comes from the externalities of multi-market simultaneous search. In each market and in the aggregate level, we have constant returns to scale in matching. Another novel feature is how we consider the transition of equilibria. When we apply the theory to understand the Great Recession, the switch of equilibrium is not due to changes of belief or coordination, but because negative financial shocks kills the LIE while the HIE becomes the unique equilibrium during the crisis.

This paper also contributes to our understanding of the slow recovery of the labor market *after* the Great Recession. There exists three main explanations for the "jobless recovery": mismatch, financial shocks and wage rigidity (real or nominal). The mismatch hypothesis says that idle workers are seeking employment in sectors, occupations, industries, or locations different from those where the available jobs are. Sahin et al. (2014) find that mismatch, across industries and three-digit occupations, explains at most one-third of the total observed increase in the unemployment rate. Lazear and Spletzer (2012) find that although mismatch increased during the recession, it retreated at the same rate. Regarding financial shocks, Chen et al. (2012) and Jermann and Quadrini (2012) study how financial shocks affect macroeconomic variables through traditional channels of friction, whereas Monacelli et al. (2011) model how financial conditions affect firms' bargaining position with workers. However, these studies cannot explain the small job finding rate, thus the jobless recovery after financial conditions improve. The third candidate explanation is about wage rigidity. Shimer (2012) studies how real wage rigidities cause jobless recoveries. Schmitt-Grohé and Uribe (2012) argue nominal rigidity and lack-of-confidence shocks help explain the jobless recovery. Bils et al. (2014) propose a model in which if wages of matched workers are stuck too high in a recession, then firms will require more effort, lowering the value of additional labor and reducing new hiring. These theories can accommodate many previous US experience whereas miss out on the prolonged recovery of the labor market after the Great Recession and the breakdown in the Beveridge curve. What we have shown in this paper is how search intensity and unemployment rate can be permanently higher after temporary financial shocks. We also use empirically realistic parameters to generate the unemployment rate observed during and after the crisis.

The rest of the paper is organized as follows. Section 2 introduces the static model, Section 3 studies the dynamic version of the model. Section 4 looks at the calibration and simulation and Section 5 concludes.

2 The Static Model

In this section we present a static model of two markets that introduces the basic assumption and illustrates the mechanism of multi-market simultaneous search.

2.1 Environment

Consider a static environment where a given measure of unemployed workers are looking for jobs. Specifically, assume there are two types of unemployed workers—Type 1 and Type 2. The measure of each type is normalized to u_i, $i \in \{1, 2\}$. There are also two markets, namely Market 1 and Market 2. For concreteness, one can think of a market as a geographical location[2]. Workers' utility is linear and they each produce y if employed. If a worker has no offer at all, she would receive the unemployed benefit, y_u, which is

[2]Other interpretations of a market could be an occupation, an industry and a sector.

smaller than y. If employed, a worker would receive a wage that is determined by bargaining. Workers of type i can either choose to only search in Market i with no cost, or to search simultaneously in both markets with additional cost c_u. In addition, they need to pay an additional cost c if employed in Market j. Such cost can represent a direct cost of relocation (including the realized capital loss when selling a house, which is very important during the fall of housing prices), or preference over markets. (We say Market i is the Type i workers' own market.)

Since workers might search in both markets, the measure of applicants in Market i, which we denote by n_i, might be greater than u_i. Let v_i be the measure of vacancies in Market i. In each Market, given the measure of vacancies and applicants, v and n, there are $m(v,n)$ of successful offers extended to applicants. An applicant (firm) in a market at most would receive (extend) one offer in that market. Assume $m(v,n)$ is increasing in both arguments, concave, constant-return-to-scale and $m(v,n) \leq \min\{v,n\}$. Let the market tightness be $\theta = v/n$ for a market, then every firm has a chance of $m(v,n)/v = m\left(1, \frac{n}{v}\right) = M(\theta)$ for extending an offer and every applicant has a chance of $m(v,n)/n = \theta M(\theta)$ for receiving an offer in the market. On the firms' side, the cost of creating a vacancy is c_v in either market. We assume free entry, which drives firms' profit to zero. Note that $M(\theta)$ is a decreasing function of θ, whereas $\theta M(\theta)$ is an increasing function of θ. Of course, both $\theta M(\theta)$ and $M(\theta)$ must be positive but smaller than one.

We need to discuss a little bit about the matching function. Economists have been using matching functions to bypass the complicated distribution of offers and the search process. Inside the black box of the matching function, a worker in fact might receive many offers in a market, some of which are rejected. If the matching function gives a worker a match, this means the worker have at least one acceptable offer in this market, which he would accept if he does not have offers from other markets. What multi-market simultaneous search brings up to the table is the possibility of offers from other markets. Moreover, it is important to note that by paying additional cost c_u to search simultaneously in other markets, a worker's chance of receiving offers from the original market should not be directly affected. To sum up, if one only searches in one market, then she finds a job unless she does not receive a match in the market; if one searches in both markets, then she finds a job unless she does not receive a match in either markets. Mathematically, the job finding rate, λ_i, for a type i worker can be written as:

$$\lambda_i = 1 - [1 - \theta_i M(\theta_i)]\,, \text{ without simultaneous search,} \tag{1}$$

$$\lambda_i = 1 - [1 - \theta_i M(\theta_i)]\left[1 - \theta_j M(\theta_j)\right]\,, \text{ with simultaneous search,} \tag{2}$$

where $i \neq j$. For an individual worker, simultaneous search reduces the chance that she cannot find a job. But it also comes with cost c_u. Therefore we can write the net surplus of simultaneously search for a Type i worker as

$$\Gamma^i = -c_u + \theta_j M(\theta_j)[1 - \theta_i M(\theta_i)](w_j - c), \tag{3}$$

where $i \neq j$ and w_j is the wage in Market j. This expression captures the idea that an offer from the other market is only useful if no offer from one's own market is received, which happens with $1 - \theta_i M(\theta_i)$ probability. Because the key decision (and actually the only decision) that workers must make is whether to simultaneously search in the other market, our focus would be on the property of the Γ function. Given $\rho y > c$, which will be clear below, simultaneously search is only profitable if and only if $\Gamma^i > 0$.

For wage determination, we assume Nash bargaining with bargaining power ρ of workers. In the bargaining, we assume that the cost c is not contractable. In other words, bargaining happens after workers have paid the cost c. This assumption is especially reasonable if the environment is dynamic and bargaining happens every period so that the cost c is sunk. The wage in a bargaining must maximize the following Nash product:

$$\max_w (w - y_u)^\rho (y - w)^{1-\rho}, \tag{4}$$

so we have a wage equation:

$$w = \rho y + (1 - \rho) y_u. \tag{5}$$

This wage equation is not affected by market tightness. Because we assume workers produce the same output in either market, the wages from both markets are the same. There are three possibilities. A worker may receive (a) only one offer from her own market; (b) only one offer from the other market; and (c) one offer from each market. Since $y > y_u$, so $w > y_u$, which means the offer in (a) will be accepted for sure. In order to make the offer in (b) acceptable, we need $\rho y > c$, which guarantees that $w - c > y_u$. This is a necessary condition for simultaneous search: one only search in other markets if the offers from there is acceptable. Intuitively, if the relocation cost is too high, nobody would simultaneously search in other markets. Next consider case (c), which is only possible if workers simultaneously search in many markets. Now it only matters if c is positive. If $c > 0$, then she would choose the offer in her own market for sure. If $c = 0$, then she is indifferent between the two offers so we assume that she choose either offer with half the chance.

Technically, each type of workers can choose whether to search in both markets, so we have four types of pure strategy equilibria . However, we only consider the following two pure strategy equilibria: a low intensity equilibrium (LIE) where both types only search in their own market, and a high intensity equilibrium (HIE) where both types search in both markets. For one thing, these two are especially important if we focus on the case of $u_1 = u_2$, so it is natural to assume both types of workers would take the same strategy. For another, once we lay out our analysis of these two equilibria, the other two are similar. In both HIE and LIE, it is obvious that $\theta_1 = \theta_2$, since all workers choose the same strategy. Therefore we use θ_L and θ_H to denote the market tightness in LIE and HIE. Given that wage is determined by (5), then the net surplus of simultaneous search, Γ, is a function of of the market tightness. LIE then requires that $\Gamma(\theta_L) < 0$ and HIE requires that $\Gamma(\theta_H) > 0$. Potentially, we can also have mixed strategy equilibrium, which requires a market tightness such that $\Gamma(\theta) = 0$. That extension is straightforward and is not discussed in this paper.

To close the model, we need to specify the labor demand by firms. Interestingly, now the free entry condition of firms depends on the search behavior of the workers. In other words, it matters what kind of equilibrium we are in. Since the firm entry condition depends on whether c is positive, we start by considering the case where $c = 0$.

2.2 Symmetric Equilibria with $c = 0$

When $c = 0$, we assume workers receiving offers from both markets choose randomly. Specifically, they choose either one with half of the chance. Now the free entry condition for firms are given by the following two equations:

$$c_v = M(\theta_L)(y - w), \text{ in LIE}; \tag{6}$$

$$c_v = M(\theta_H)(y - w)\left\{[1 - \theta_H M(\theta_H)] + \frac{1}{2}\theta_H M(\theta_H)\right\}, \text{ in HIE}. \tag{7}$$

The first equation is straightforward: the expected gains from the vacancy is just enough to cover the cost of creating the vacancy. In the second equation, the idea is the same, although the third term on the right-hand side is worth some discussion. Given a worker receives an match in this market, with $1 - \theta_H M(\theta_H)$ probability she does not have an offer from the other market so she is sure to accept the job in this market, whereas with $\theta_H M(\theta_H)$ probability she also receives an acceptable offer from the other market, so that she only accepts the offer in this market with half of the chance.

Because both $M(\theta)$ and $M(\theta)\left[1 - \frac{1}{2}\theta M(\theta)\right]$ are monotonically decreasing in θ, the above two equations pin down the two levels of market tightness. We can also infer that the offer extension rate $M(\theta)$ in HIE is higher so that the probability that firms actually hire a worker is the same. We must have $\theta_H < \theta_L$. In other words, in HIE, firms must be compensated by the lower market tightness for the fact that now offers may be rejected. Lower market tightness also means $\theta_H M(\theta_H) < \theta_L M(\theta_L)$, or to say, the matching rate in a market for an application is lower than that in the LIE. However, what happens to the total chance of finding a job, which include the chance of finding jobs in both market is not so obvious. For that one can look at (1) and (2). We can see that although $[1 - \theta_i M(\theta_i)]$ is higher in HIE, it is also multiplied by $\left[1 - \theta_j M\left(\theta_j\right)\right]$ which is smaller than 1. Whether $\lambda_L > \lambda_H$ is true would depend on the matching function and the value of θ_L and θ_H. For these values of θ_L and θ_H we can then check if the optimal strategy of workers are consistent with these market tightness. Specifically if $\Gamma(\theta_L) < 0$ then we have LIE; and if $\Gamma(\theta_H) > 0$ then we have HIE. Note next that the $\Gamma(\theta)$ can be written as

$$\Gamma(\theta) = -c_u + \left\{ -\left[\theta M(\theta) - \frac{1}{2}\right]^2 + \frac{1}{4} \right\} [\rho y + (1-\rho) y_u]. \tag{8}$$

Then obviously we have the following two cases:

- Case 1. If $c_u / [\rho y + (1-\rho) y_u] > \frac{1}{4}$, then $\Gamma(\theta)$ is always negative so that only LIE exist.
- Case 2. If $c_u / [\rho y + (1-\rho) y_u] \in (0, \frac{1}{4}]$, then there exist two cutoffs $\underline{\theta}$ and $\bar{\theta}$, with $\underline{\theta} \leq \frac{1}{2} \leq \bar{\theta}$, such that $\Gamma(\underline{\theta}) = \Gamma(\bar{\theta}) = 0$. Furthermore, if $\theta \in [\underline{\theta}, \bar{\theta}]$, then $\Gamma(\theta) \geq 0$, and otherwise, $\Gamma(\theta) < 0$.

In the second case, the θ space is divided into three regions. Because $\theta_L > \theta_H$, we then have six combinations of the position of θ_L and θ_H. For example, if both of them are in the region of $(\underline{\theta}, \bar{\theta})$, then both $\Gamma(\theta_L)$ and $\Gamma(\theta_H)$ are positive and so only HIE is feasible. If $\theta_L \in (\underline{\theta}, \bar{\theta})$ and $\theta_H < \underline{\theta}$, then $\Gamma(\theta_L) > 0$ and $\Gamma(\theta_H) < 0$, then no symmetric pure strategy equilibrium exists.

Multiple Equilibria

~~Most~~ interestingly, multiple equilibria are ~~also~~ possible. Mathematically this happens if $\theta_H \in (\underline{\theta}, \bar{\theta})$ and $\theta_L > \bar{\theta}$. Figure 2 shows such an example of multiple equilibria. But what is the intuition? We all know that with free entry and constant-return-to-scale matching function, if workers always only search in one market, then moving one worker from one market to another would not cause the job finding rate to change in either market. But with the possibility of multi-market simultaneous search, things are different. Suppose we start with LIE, and then let a worker of Type 1 simultaneously search in Market 2. This poses externalities to two groups of people. First, she lowers Type 2 workers' chance

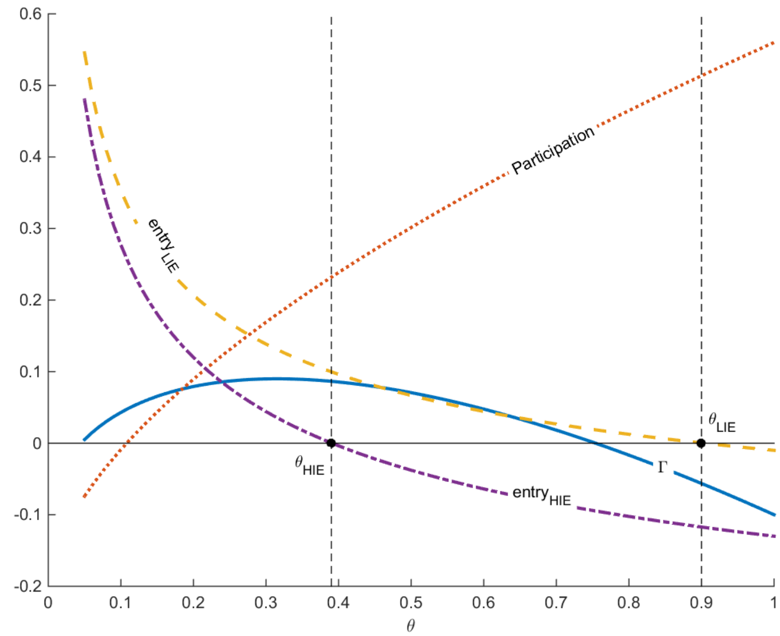

Figure 2: Simulation of the static model

of finding jobs in Market 2, because sometimes she receives an offer from each market and have to reject one of them. These rejections of offers do not happen when everyone only searches in their own markets. Firms in Market 2 must be compensated by a lower market tightness, which causes the job finding rate in Market 2 to fall. Then workers originally only search in Market 2 now find it more attractive to simultaneously search in Market 1, because in (3) we see that as one finds it hard to find jobs in her own market, i.e. $\theta_i M(\theta_i)$ decreases, the net surplus of simultaneous search increases. Again, as mentioned above, simultaneous search is only useful if one cannot find a job in one's own market. Another group of agents are also affected: her own type of workers, since simultaneous search also increases her chance of rejecting offers in her own market. The result is that both types of workers are negatively affected by her simultaneous search, making them more likely to simultaneously search, as well. Therefore workers have strategic complementarity in choosing search intensity. Then whether we have multiple equilibria or not simply depend on the magnitude of such complementarity, i.e. is it enough to overcome the additional search cost, c_u.

Many paper have strategic complementarity and multiple equilibria because of increasing-return-to-scale production function or matching function. But in our approach, both the market specific matching function $m(v, n)$ and the implied aggregate matching function are constant return to scale. The job finding rates of both equilibria are determined by market tightness. Our theory is consistent with the empirical evidence of constant-return-to-scale aggregate matching function.

Welfare: If we have multiple equilibria, can we rank LIE and HIE according to efficiency? Since firms earn zero profit, the welfare is completely summarized by the job finding rate and how many workers paid the relocation cost if $c > 0$. From Equations (1), (2), (6), and (7), we have $\lambda_H/\lambda_L = 2\theta_H/\theta_L$. This means the job finding rate in the HIE is lower than that in the LIE if and only if $2\theta_H < \theta_L$. This looks like a drastic change in the observed market tightness. It is not. One has to note the definition of θ: v/n, which is the same as v/u in the LIE and one half of v/u in the HIE. The condition $2\theta_H < \theta_L$ is the same as $v_H/u_H < v_L/u_L$, which means as long as the observed market tightness in the HIE is smaller than that in the LIE, then the job finding rate is lower in the HIE. Because the society pays c in the HIE but not in the LIE, this condition is also a sufficient condition for the HIE to be less efficient than the LIE.

Financial Shocks: Given the massive labor market consequences after the 2008 financial crisis, an important application of our theory is to think about the effects of financial shocks on the labor market. Because the process of creating vacancies can be thought of investing now and receiving cash flow in the future, the entry cost c_v is naturally affected by what happens in the financial market. During financial crisis, it would be hard for firms to borrow, raising their financing cost. So we interpret an increase in c_v as a negative financial shock.

Take Figure 2 as an example, if we have higher c_v due to negative financial shocks, then both entry curves would shift to the left. That is, with higher entry cost, then the market tightness needs to be smaller in order for firms to make even. But if the shock is big enough, then θ_L would lie in the region where Γ is positive. But if that is the case, LIE would no longer be an equilibrium. This highlights the instability of the economy if we allow for multi-market simultaneous search. In fact, in Section 4 we will construct examples where we are initially at the LIE but large enough financial shocks can cause the economy to permanently switch equilibrium to the HIE.

Productivity Shocks: We can interpret a decrease in y as a negative productivity shock in our model. Will productivity shocks have similar effects as financial shocks? Take Figure 2 as an example, again. It is true from the two firm entry conditions (6) and (7), if y decreases, then the two entry curves in Figure 2 would both shift to the left, the same as with negative financial shocks. But the Γ curve shifts downward, as well. In fact, we might never be able to get rid of the LIE, i.e. $\Gamma(\theta_L)$ could always be negative. This means, financial shocks are more likely to cause structural changes in the labor market than productivity shocks.

2.3 Symmetric Equilibria with $c > 0$

Before we discuss the details of this subsection, we want to point out that the size of u_1 and u_2 does not play any specific role in determining the equilibrium in Section 3.2. Especially the free entry condition for HIE does not depend on the size of u_1 and u_2. The condition $u_1 = u_2$ is only useful in justifying our assumption of symmetric equilibrium. Now if $c > 0$, then it means that workers have some preference over markets. Thus the sizes of u_1 and u_2 matter. The free entry condition for LIE is the same as (6), but that for HIE (in Market i) is different now:

$$c_v = M(\theta_i)(y - w)\left\{\left[1 - \theta_j M(\theta_j)\right] + \frac{u_i}{u_i + u_j}\theta_j M(\theta_j)\right\}, \text{ in HIE.} \tag{9}$$

The interpretation is a bit different now: Given a worker receives a match in Market i, with $1 - \theta_j M(\theta_j)$ probability she does not have an offer from the other market so she is sure to accept the job in Market i, whereas with $\theta_j M(\theta_j)$ probability she also receives offers from Market j and then the offer in Market i would be accepted if and only if it is extended to a Type i worker. Note that now when a worker simultaneously search in the other market, she only imposes negative externality to the other type of workers. Moreover, if u_1 and u_2 are very different, then it would be important to consider the equilibria where one type of workers only search in one market and the other type of workers search in both.

However, if $u_i = u_j$, then the entry condition of (9) is the same as (7). In the previous subsection, we assume that if a worker receives offers from both markets, offers are rejected with half of the chance. Here, as long as $u_1 = u_2$, then that chance is also exactly $1/2$. Since our main focus is symmetric equilibrium with $u_1 = u_2$, the other parts of analysis is similar so we skip those here.

Before we move on, we need to discuss one more issue: one might expect the employers to treat the two types of workers differently. Because with positive relocation costs, "local" workers, who would not incur this cost, are more likely to accept an offer in the local market. But in reality this could be hard. It requires that employers in Market i can tell if a worker from Market j is searching simultaneously or if she is genuinely moving to Market i and thus only looks for jobs in Market i, possibility due to some personal reasons. Any job candidate can claim the latter. Given the asymmetric information, the employers might choose to statistically discriminate against one type of workers. So a more realistic setup could be to assume that when type i workers apply for jobs in Market j she has less search efficiency units, i.e. she counts as a fraction of an applicant in the matching function (e.g. her applications in Market j sometimes get lost or overlooked). But as long as these simultaneous searchers sometimes receive offers from both markets, the strategic complementarity exists and so does the possibility of multiple equilibria.

2.4 A N-Market Environment

We deliberately use the two-market setup so as to make it simple but still enough to illustrate the economics of multi-market simultaneous search. We would like to further point out now that with some slight modifications, the economics AND the mathematical equations are the same in a N-market environment. Suppose $c = 0$ and there are N markets and N types of workers. A worker from market i can choose one of the following two options: to search only in the local market, or to search both the local market and randomly one of the remaining $N-1$ markets. When a worker does multi-market simultaneous search if we assume the probability of searching in the nth ($n \neq i$) market is proportional to the size of the market, i.e. u_n, then it is as if for every market i there is a mirror market i' that operates just like the other market in the two-market setting. The math is the same.

2.5 Mixed Strategy Equilibrium

This again is not the focus of this paper, we only briefly describe how one should think of mixed strategy equilibrium in this model. First, a mixed strategy equilibrium requires that the net surplus of simultaneous search is zero so that workers are indifferent between the two strategies, i.e. $\Gamma = 0$. For example, in Figure 2, we have two values of θ that can make $\Gamma = 0$: one around 0.75 and the other around 0.05. Think first about the case of zero relocation cost, i.e. $c = 0$. Suppose in a mixed strategy equilibrium (thereafter MSE) there are ζ fraction of workers adopt the simultaneous search strategy. Let θ_M be the market tightness of the mixed strategy equilibrium [so $\Gamma(\theta_M) = 0$], then the entry condition can be written as

$$c_v = M(\theta_M)(y-w)\left\{[1-\zeta\theta_M M(\theta_M)] + \frac{1}{2}\zeta\theta_M M(\theta_M)\right\}, \textit{ in MSE.}$$

The third term on the right-hand-side is a convex combination of the third term of the right-hand-side in (6) and that in (7). As long as θ_H lies to the left of θ_M and θ_L lies to the right of it, we are sure that we have a MSE, such as the case in Figure 2. Notice that the MSE does not require the existence of multiple equilibria or even the existence of pure strategy equilibrium. For example, if θ_H is smaller than 0.5 in Figure 2, then we have the LIE and two MSE's. If both θ_H and θ_L are above 0.75, then we only have the LIE. If $\theta_L \in (0.05, 0.75)$ and $\theta_H < 0.05$, then the only equilibrium is the MSE. The rest of the paper would focus on the pure strategy equilibria and leave MSE to future studies.

3 The Dynamic Model

The basic economics of our theory has been laid out in the static version of the model in the previous section. We have learned that workers' multi-market simultaneous search behavior can impose negative externality to other workers, because this increases the rejection rate of the offers extended by by firms, lowering their incentive to create vacancies (they require a higher vacancy/applicant ratio to make even). The negative externality causes strategic complementarity among workers and makes possible multiple equilibria. We have also learned that the stability of equilibria is more susceptible to financial shocks than to productivity shocks.

Next, we extend the static model into a dynamic environment. The economics is the same, but it is important to have a dynamic version in order to show the dynamic effects of financial shocks and to conduct quantitative exercises. As Section 3.2 and 3.3 show, the mechanism of the model works with or without a positive relocation cost, c. Since we later interpret different markets as geographical locations in our calibration, it is natural to have a positive relocation cost. So in this section we introduce a two-market dynamic version of the model where the cost c is paid one time[3]. With this dynamic setting, if an agent has moved to a new market and paid the cost c, then she belongs to this new market. If she later loses her job and finds another job in the original market, then she needs to pay the relocation cost, again.

3.1 Model Setup

Workers' Value Function

For an employed worker, there is no active action needed. The value function for the employed in Market i can be written as:

$$W_t^i = w_t + \beta\delta E_t\left(U_{t+1}^i\right) + \beta\left(1-\delta\right)E_t\left(W_{t+1}^i\right), \tag{10}$$

[3] Alternatively one can assume some cost is paid every period: such as moving from somewhere one really likes to somewhere one really dislikes, so that each period she incurs some cost. But the math and economics is similar to a one time cost paid.

where β is the common discount factor, δ the exogenous probability that a worker and his job are separated in a period, and U the value function of an unemployed worker. There are two choices in the action space of an unemployed Type i worker: applying for job in market i only, or applying for jobs in both markets.[4] The value function for an unemployed Type i worker is

$$U_t^i = y_u + \beta\theta_{it} M(\theta_{it}) E_t\left(W_{t+1}^i\right) + \beta\left[1 - \theta_{it} M(\theta_{it})\right] E_t\left(U_{t+1}^i\right) + \max\left(0, \Gamma_t^i\right). \quad (11)$$

This value function looks the same as in the standard one-market search model except for the max operator of the last term. Inside the max operator is the net surplus of simultaneous search:

$$\Gamma_t^i = \beta\theta_{jt} M\left(\theta_{jt}\right)\left[1 - \theta_{it} M(\theta_{it})\right]\left[E_t\left(W_{t+1}^j\right) - E_t\left(U_{t+1}^i\right) - c\right] - c_u \quad (12)$$

In other words, if $\Gamma_t^i < 0$, then the unemployed only search in their favorite market, in period t; and if $\Gamma_t^i > 0$, then the unemployed search in both markets. We have implicitly assumed that the difference of $E_t\left(W_{t+1}^j\right)$ and $E_t\left(W_{t+1}^i\right)$ is smaller than c,[5] so that Type i agents, upon receiving offers from both markets, will accept the offer from market i.

(Next, suppose at the beginning of period t a previously employed worker face a separation shock, she is in the unemployed pool this period, so can only get y_u for consumption. Even if she immediately find a match, she will start working in the next period. It is possible for a newly employed worker to lose job and get back to the unemployed pool immediately. But this worker gets to produce at least one period after she accepts the offer.)

Firms' Value Function

The firm decides how many vacancies to create by comparing the cost and the benefit of opening a vacancy. The cost of opening a vacancy is given by c_v. The benefit of opening a vacancy is given by the product of the probability of filling a vacancy, and the present discounted value of the profits generated by an additional employee, J_t. Like in the static

[4]We are interested in the symmetric equilibrium where identical agents take identical actions. If an unemployed type-i worker applies for market $j \neq i$ job only, then applying for market i job only will be a profitable deviation.

[5]Theoretically, there could be cases where $E_t\left(W_{t+1}^j\right) - c \geq E_t\left(W_{t+1}^i\right)$, which suggest that being employed in market j is much better than being employed than in Market i. It can happen temporarily, for example, when one market is hitter by recession more than the other market. But as workers flow to Market j, the difference would gradually decrease. (We may pursue this idea if we simulate differential shocks to the two markets.)

model, we assume that firms operate a constant return to scale technology, so the value of an additional employee to the firm is independent of the number of workers employed by the firms and, hence, the firm's problem is linear. The value of a filled vacancy to a firm is given by

$$J_t = y_t - w_t + \beta (1 - \delta) E_t (J_{t+1}) \tag{13}$$

Entry Condition and Law of Motion of Unemployment

As in the static version, the entry condition for firms depends on workers' search behavior. Again, we only consider pure strategies by the workers and assume symmetric equilibrium and that the two markets are of similar size so that all workers make the same search choice. Technically, an equilibrium is a sequence of search choices that are consistent with workers' value functions and other equilibrium conditions to be laid out. For example, workers search in both markets in every even-numbered periods and in only one market in every odd-numbered periods is an equilibrium candidate. Now we would like to use the LIE to denote the equilibrium in which all unemployed workers always search in their local markets and the HIE the equilibrium in which all unemployed workers always search in both markets. We continue to assume we are either in the LIE, or in the HIE. Then similar to (6) and (7), the corresponding entry conditions in Market i can be written as

$$c_{vt} = \beta M(\theta_{it}) E_t (J_{t+1}), \textit{ in the LIE}; \tag{14}$$

$$c_{vt} = \beta M(\theta_{it}) \left[1 - \frac{u_{jt}}{u_{it} + u_{jt}} M(\theta_{jt}) \right] E_t (J_{t+1}), \textit{ in the HIE}, \tag{15}$$

where now the entry cost c_{vt} can vary over time due to financial shocks and u_{it} and u_{jt} are endogenously determined. The law of motion for unemployment rate for Type i workers are as follows ($i \neq j$):

$$u_{i,t+1} = u_{it} + (1 - u_{it}) \delta - u_{it} \theta_{it} M(\theta_{it}), \textit{ in the LIE}, \tag{16}$$

$$u_{i,t+1} = u_{it} + (1 - u_{it}) \delta - u_{it} \left\{ \theta_{it} M(\theta_{it}) + \theta_{jt} M(\theta_{jt}) [1 - \theta_{it} M(\theta_{it})] \right\}, \textit{ in the HIE}. \tag{17}$$

In both (16) and (17), the third term on the right-hand-side are the unemployment rate times the job finding rate in that equilibrium. Again, we can show that the job finding rate in HIE is lower than LIE if $2\theta_H < \theta_L$.

Wage Determination

We assume a matched worker and her firm bargain over wage every period. Worker's and firm's outside options are as follows. The outside option for the worker is to take the unemployment income y_u and enter the next period still matched with the same firm; the outside option for the firm is to produce nothing and enter the next period still matched with the same worker. In such a game, the wage outcome is same as (5) in the static game. The outside options here certainly simplify the analysis compared to Pissarides (1985), Mortensen and Pissarides (1994) and many subsequent papers. But as argued by Hall and Milgrom (2008), the threat to terminate the relationship is irrational for both parties and the credible threat point in the bargain is to delay bargaining. Our wage outcome is consistent with this view.[6]

3.2 Steady State Equilibria

In steady state, c_{vt} is constant over time, so are J_t, θ_{it}, W_t^i, W_t^j, U_t^i and u_{it}, where $i \neq j$ and $i,\ j = 1$ or 2. Here we only discuss the case where the two markets are identical. From (13), we can write $[1-\beta(1-\delta)]\,J = y - w$. The entry conditions (14) and (15) thus uniquely pinned down the market tightness in each type of equilibria. We use θ_L and θ_H to denote the market tightness in the two types of equilibria. Using subscript s to stand for H or L, from (10) and (11), we have

$$(1-\beta)\,W_s = w - \beta\delta\,(W_s - U_s)\,, \tag{18}$$

$$(1-\beta)\,U_s = y_u + \beta\theta_s M(\theta_s)\,(W_s - U_s) + \max\left[0, \Gamma_s(\theta_s)\right]. \tag{19}$$

Subtract (19) from (18) we can derive the following:

$$W_s - U_s = \frac{w - y_u - \max\left[0, \Gamma_s(\theta_s)\right]}{1-\beta+\beta\delta+\beta\theta_s M(\theta_s)}, \tag{20}$$

$$(1-\beta)\,U_s = \frac{\beta\theta_s M(\theta_s)\,w + (1-\beta+\beta\delta)\left\{y_u + \max\left[0,\Gamma_s(\theta_s)\right]\right\}}{1-\beta+\beta\delta+\beta\theta_s M(\theta_s)} \tag{21}$$

$$(1-\beta)\,W_s = \frac{\left[1-\beta+\beta\theta_s M(\theta_s)\right]w + \beta\delta\left\{y_u + \max\left[0,\Gamma_s(\theta_s)\right]\right\}}{1-\beta+\beta\delta+\beta\theta_s M(\theta_s)} \tag{22}$$

[6] Similar wage outcome is used in Kaplan and Menzio (Forthcoming).

From (20) it is apparent that if the term $\max\left[0, \Gamma_s\left(\theta_s\right)\right]$ is higher, it is as if the unemployed has a higher y_u. Also, both U_s and W_s are weighted average of w and $y_u + \max\left[0, \Gamma_s\left(\theta_s\right)\right]$. For U_s, the weight of w is given by the probability of finding a job in the next period. We have assumed that $W_i - U_i > c$, otherwise simultaneous search is never optimal, even when c_u is zero and no matter what the job finding rate is.

Low Intensity Equilibrium

In the LIE, the market tightness is given by (14), which in steady state can be written as $c_v = \beta J M\left(\theta_L\right)$. Of course, we need the incentive condition: $\Gamma_L\left(\theta_L\right) < 0$. The steady state version of (12) is straight forward. We can see now that c_u does not directly affect the values of W_L and U_L, but affect the incentive condition of (12). Lastly, using (16) the unemployment rate in steady state is given by

$$u_L = \frac{\delta}{\delta + \theta_L M\left(\theta_L\right)} \tag{23}$$

High Intensity Equilibrium

In the HIE, the market tightness is given by (15) which in steady state can be written as (we assume the two markets are identical now):

$$\frac{c_v}{\beta J} = M\left(\theta_H\right)\left[1 - \frac{1}{2}\theta_H M\left(\theta_H\right)\right], \text{ in HIE.} \tag{24}$$

Similar to the LIE, we also need the incentive condition: $\Gamma_H\left(\theta_H\right) > 0$. Using (20), we have[7]

$$\Gamma_H\left(\theta_H\right)\left[1 + \chi\right] = \beta\theta_H M\left(\theta_H\right)\left[1 - \theta_H M\left(\theta_H\right)\right]\left[\frac{w - y_u}{1 - \beta + \beta\delta + \beta\theta_H M\left(\theta_H\right)} - c\right] - c_u, \tag{25}$$

where the right-hand-side is $\Gamma_L\left(\theta_H\right)$, that is, the net surplus of simultaneous search in LIE evaluated at θ_H. Because $\chi > 0$ and the above equation holds true for any number of θ_H, we know that $\Gamma_H\left(\theta\right)$ and $\Gamma_L\left(\theta\right)$ always have the same sign. Lastly, using (17) and the fact that the two markets are identical, the unemployment rate in steady state is given by

$$u_H = \frac{\delta}{\delta + \theta_H M\left(\theta_H\right)\left[2 - \theta_H M\left(\theta_H\right)\right]} \tag{26}$$

[7] here $\chi = \frac{\beta\theta_H M(\theta_H)[1-\theta_H M(\theta_H)]}{1-\beta+\beta\delta+\beta\theta_H M(\theta_H)}$

Comparing LIE and HIE

Similar to the static model, we have $\theta_L > \theta_H$. And because of the two entry conditions $c_v = \beta J M(\theta_L)$ and (24), we have

$$M(\theta_L) = M(\theta_H)\left[1 - \frac{1}{2}\theta_H M(\theta_H)\right]$$

$$u_H = \frac{\delta}{\delta + \theta_H M(\theta_H)\left[2 - \theta_H M(\theta_H)\right]} = \frac{\delta}{\delta + 2\theta_H M(\theta_L)}$$

Compare it with (23), we know that $u_H > u_L$ if and only if $\theta_H \leq \theta_L/2$. This is the condition that tells us when the HIE would generate a higher unemployment rate than the LIE. In this case, the job finding rate is also lower in the HIE:

$$\theta_H M(\theta_H)\left[2 - \theta_H M(\theta_H)\right] \leq \theta_L M(\theta_L)$$

Intuitively, HIE is less efficient than LIE if and only if the market tightness in HIE is sufficiently lower than LIE. In other words, HIE is less efficient if the negative externality caused by simultaneous search is sufficiently strong.

4 Calibration and Simulation

In this section, we calibrate the model to match features of the US economy. We show that the calibrated model features multiple equilibria, and a small shock is able to cause a much larger, self-fulfilling and persistent changes in the labor market.

4.1 Calibration Strategy

The property and behavior of the model is decided by a total of 11 parameters. The value of these parameters is chosen such that when evaluated at the steady state with the lowest unemployment rate, the calibrated economy matches features of the US economy between 1987 and 2007, as in Kaplan and Menzio (Forthcoming). We normalize a time period to be one month.

Without loss of generality, we normalize the productivity of a matched worker-firm pair to $y = 1$, and the unemployment benefit to $y_u = 0.2$, which lies in the range of $[0, 0.4]$, as indicated in Shimer (2005). We set the discount factor to $\beta = 0.997$, which is equivalent to an annual discount factor of 0.965.[8] The exogenous separation rate, $\delta = 0.024$, is chosen

[8] This is derived from a time discount rate of 0.003, as used in Kaplan and Menzio (Forthcoming),

to match the average monthly transition rate from employment to unemployment in the U.S. between 1987 and 2007. We set the worker's bargaining power to $\rho = 0.74$.[9]

We choose a CES matching function,[10]

$$M(u,v) = \kappa \left(u^{-\phi} + \alpha v^{-\phi}\right)^{-\frac{1}{\phi}} \tag{27}$$

A special case of this matching function is used in Kaplan and Menzio (Forthcoming) by setting $\kappa = \alpha = 1$. When $\phi = 0$, it becomes a Cobb-Douglas matching function, as used in many literature (e.g., Shimer, 2005). We set the parameter α in the matching function to $\alpha = 0.389$. This is calculated as the ratio of the power terms in the Cobb-Douglas matching function, as in Shimer (2005).[11] In Appendix A, we let α vary between 0.3 and 0.7 as robustness check.

The matching function parameters of constant and elasticity, κ and ϕ, are chosen to match the elasticity of the job finding rate with respect to the market tightness and the average v-u ratio (aka the market tightness). Specifically, we have

$$\phi = -\frac{\ln\left(\frac{\hat{\eta}}{\alpha(1-\hat{\eta})}\right)}{\ln\hat{\theta}}, \text{ and } \kappa = f\left(\hat{\theta}\right)\left(1-\hat{\eta}\right)^{-\frac{1}{\phi}} \tag{28}$$

where $\hat{\eta}$ and $\hat{\theta}$ are the targeted values of the elasticity and the v-u ratio, respectively, and $f\left(\hat{\theta}\right)$ is the job finding rate.[12] As noted in Menzio and Shi (2011) and Kaplan and Menzio (Forthcoming), after taking into account the on-the-job search, the the elasticity of the job finding rate with respect to the market tightness is 65% in the data, which is our targeted

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β=exp (-0.003)= 0.997.

[9] It is same as in Kaplan and Menzio (Forthcoming). Shimer (2005) chooses $\rho = 0.72$.

[10] Note that as long as $\kappa \leq \min\left\{1, \alpha^{\frac{1}{\phi}}\right\}$, we have $f(\theta), q(\theta) \in [0,1], \forall\theta \in \mathbf{R}^{+}$, where $\theta = \frac{v}{u}$ is the measure of market tightness. This condition is always satisfied in the calibrated baseline model and the variants.

[11] In Shimer (2005), the matching function is $1.355u^{0.72}v^{1-0.72}$. We set $\alpha = (1-0.72)/0.72$.

[12] Knowing the job finding rate, $f\left(\hat{\theta}\right)$, we can back out the market tightness from the matching function,

$$\hat{\theta} = \left\{\left[\frac{f\left(\hat{\theta}\right)}{\kappa}\right]^{-\phi} - 1\right\}^{-\frac{1}{\phi}} \alpha^{\frac{1}{\phi}}$$

The elasticity of the job finding rate with respect to the market tightness is

$$\hat{\eta} = \frac{\partial \ln f(\theta)}{\partial \ln \theta}\Big|_{f\left(\hat{\theta}\right)} = 1 - \left[\frac{f\left(\hat{\theta}\right)}{\kappa}\right]^{\phi}$$

Combining these two equations, we derive (28).

value. We choose to target the v-u ratio at 0.8, which lies in the range of calculations in Hall and Schulhofer-Wohl (2015).[13] The job finding rate is measured as the average monthly transition rate from unemployment to employment, which is 0.433 in the U.S. between 1987 and 2007. Given the matching function, the vacancy costs, c_v, is chosen to match this average monthly transition rate from unemployment to employment, as in Shimer (2005) and Kaplan and Menzio (Forthcoming). We choose the worker's additional search costs, $c_u = 0.25$, and the one time moving costs, $c = 0.3$. In Appendix A, we show that the result holds for a wide range of ~~the~~ values of c_u and c.[14]

4.2 Calibration Results

Table 2 reports the normalized and calibrated parameter values. We find the calibrated model features two steady states. At the first steady state, the unemployment rate is 5.25%. At the second steady state, the unemployment rate is 9.67%, which is 84.2% higher than the unemployment rate in the first steady state.

Figure 3 illustrates the multiplicity of the equilibria in the calibrated model. If the firm expects unemployed workers to search with low intensity (search in one market only), then the firm's profit as a function of the market tightness is plotted as the dashed line, labeled as "$\text{entry}_{\text{LIE}}$". The free-entry results results that at equilibrium, if exists, the market tightness is θ_{LIE}. At this market tightness, the worker's individual rationality condition is negative, $\Gamma_{LIE} < 0$. That is, an unemployed worker searches with low intensity. Thus, θ_{LIE} is indeed an equilibrium. On the other hand, if the firm expects high search intensity by the unemployed workers (search simultaneously in both markets), then the firm's profit function is plotted as the dash-dot line in Figure 3, labeled as "$\text{entry}_{\text{HIE}}$". At θ_{HIE}, the free entry condition is satisfied; the worker's individual rationality condition is positive, $\Gamma_{HIE} > 0$, so an unemployed worker search with ~~hi~~gh intensity. This implies that θ_{HIE} is also an equilibrium. Since $\theta_{HIE} < \theta_{LIE}/2$, the resulting unemployment rate is higher in the equilibrium with high search intensity.

4.3 Response to Shocks

Standard search models (e.g., Mortensen and Pissarides, 1994; Shimer, 2005) typically has a unique equilibrium, thus all the fluctuations in the labor market generated by the

[13] In Figure 4 of Hall and Schulhofer-Wohl (2015), the v-u ratio ranges roughly between 0.7 and 0.85 in the United States between 2001 and 2008.

[14] We can not simultaneously point-identify in our model, as they together determine a worker's individual rationality constraint in the search behavior.

Table 2: Normalization and calibration outcome

Parameters		Values
Productivity of a matched pair	y	1.0
Unemployment benefit	y_u	0.2
Discount rate	β	0.997
Exogenous destruction rate	δ	0.024
Worker's bargaining power	ρ	0.74
Matching function: coef of v	α	0.389
Matching function: constant	κ	0.503
Matching function: elasticity	ϕ	7.007
Firm's vacancy costs	c_v	4.169
Worker's additional search costs	c_u	0.25
Worker's one time moving costs	c	0.35

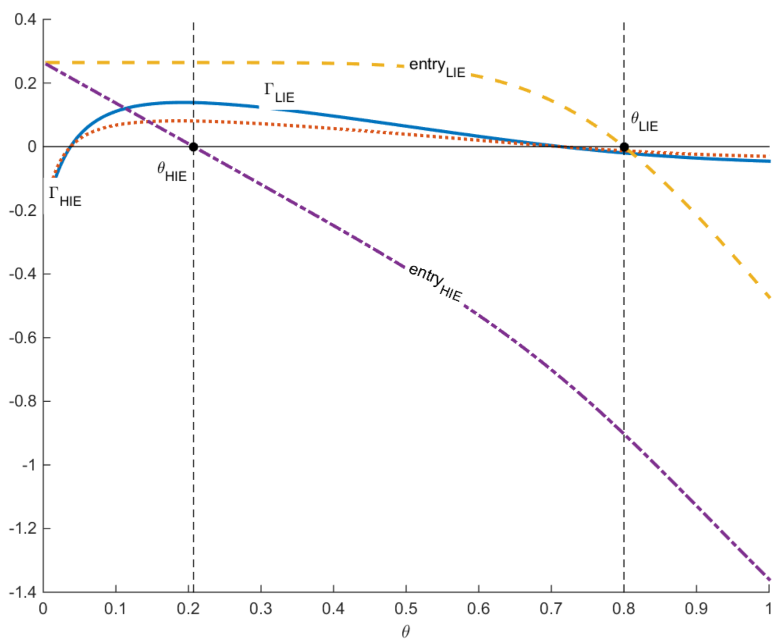

Figure 3: Multiple equilibria in the calibrated model

model come from changes in fundamental factors, such as preferences, technologies, or the environment. We have shown that our model admits multiple equilibria for empirically relevant parameter values when calibrated to the U.S. economy. This suggests that the model is capable of generating additional fluctuations in the labor market, which are caused by non-fundamental factors, such as expectations of the status of future economy. In this subsection, we show that when the agent's expectation toward the future search intensity changes, a small shock is able to cause a much larger and persistent changes in the labor market and such changes are self-fulfilling. Assume that the economy is initially in the steady state with the lowest unemployment rate, and the vacancy costs increases temporarily due to a negative financial shock which occurs between April and May of 2008.[15] The negative shock may stay for one or several periods, after which it disappears and the vacancy costs reverts to the previous level. We now simulate the dynamics of the model for this process.

Assume when the negative shock happens, the vacancy costs increases by 4% to $c_v' = 1.04c_v$. The equilibrium with low search intensity can no longer hold under this new vacancy costs. To see this, suppose that unemployed workers still search with low intensity. The entry firm's profit shifts southwest, to the solid line labeled as "$\text{entry}'_{\text{LIE}}$" in Figure 4. However, at the resulting θ'_{LIE}, the unemployed worker's individual rationality condition becomes positive and it becomes profitable for them to increase their search intensity and search in both markets. When unemployed workers search with high intensity, the firm's new profit function shifts southwest to the dashed line labeled as "$\text{entry}'_{\text{HIE}}$". At the new resulting θ'_{HIE}, the unemployed worker's individual rationality condition is positive, thus it is supported as an equilibrium. That is, at the new entry costs c_v', there exists a unique pure-strategy equilibrium where unemployed workers search with high intensity. Note that in the model we assume the switching from the initial equilibrium of low search intensity to the new equilibrium of high search intensity happens in one period, which is set to one month. In reality, however, this equilibrium switching may take much longer time to materialize.

We first assume that the vacancy costs revert to the initial value after one period, $c_v'' = c_v$, as shown in Figure 5. This is equivalent to simulate an impulse response function. The new economy features two equilibria as it is identical to the initial calibrated model: in one equilibrium unemployed workers search with low intensity and in the other equilibrium unemployed workers search with high intensity. For this reason, we simulate

[15] We assume the shock hits between April and May in 2008, when the unemployment rate starts rising in the data. This might not be the official starting date of the recent recession. However, we only focus on the labor market, so we think it is innocuous to assume such timing.

two transition paths. In the first transition path, we assume the economy stays in the new equilibrium of high search intensity even after the vacancy costs revert to the initial level and name it the HIE path; in the second transition path, we assume the economy switches back to the initial equilibrium of low search intensity and name it the LIE path. The unemployment rates along these two different transition paths are plotted in Figure 6. In both paths, the unemployment rate converges to the steady state quickly. After 12 (or 6) months, the unemployment rate is already within the $\pm 1\%$ interval of the steady state value in the first (or second) case. The individual rationality conditions for both paths are plotted in Figure 7, which shows both of them are supported as an equilibrium path.

One striking feature of the HIE path is that it matches the average unemployment rate of the actual U.S. labor market between 2009 and 2011 fairly well, as shown in Figure 6. The average unemployment rate during this period is 9.64% in the HIE path, compared to 9.50% in the actual labor market. Note that the model is calibrated to match the pre-2008 U.S. economy and we do not target the post-2008 unemployment rate. The fact that the HIE path maths with the actual unemployment rate comes directly as the feature of multiple equilibria in the model.[16] On the other hand, the unemployment rate in the LIE path decreases very quickly to the pre-shock level.

Although in theory both paths are feasible, we argue that the HIE path may be more empirically relevant for the great recession of 2008. The key difference between the HIE path and the LIE path is the unemployed worker's search behavior: they search with high intensity in the HIE path and with low intensity in the LIE path after the shock is gone. As presented in Figure 1, in the U.S. labor market, the unemployed workers spent 25% more time on job search during 2009 and 2014 than the period of 2003 to 2007.[17]

The HIE path also has the potential to explain the "jobless recovery" after the great recession.

5 Conclusion

In the current version of the paper, we analyze the unemployed workers' choice of how broadly to search for jobs, i.e., search in one market only or search in multiple mar-

[16] The unemployment rate along the HIE path rises much faster than the actual one. This is because the transitional dynamics of the model is very fast. In reality, the unemployment rate moves slower for various reasons.

[17] After the shock is gone, it also requires greater level of coordination between unemployed workers and firms to switch from the equilibrium with high search intensity to the equilibrium with low search intensity, compared to staying at the equilibrium with high search intensity. Similarly, it is not easy to switch from the LIE to the HIE either. Compared with many previous recessions, the great recession of 2008 is featured with a prolonged period of multiple negative shocks, which may trigger the switch of equilibria.

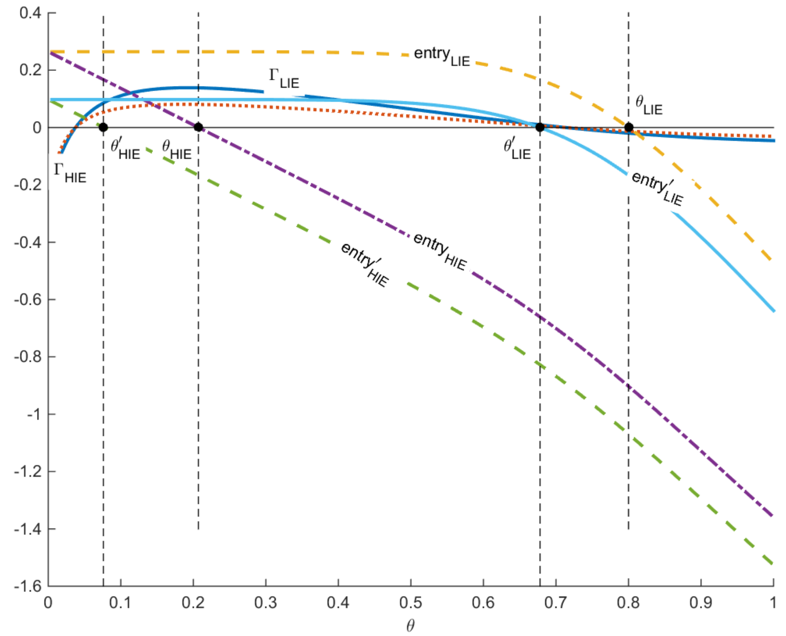

Figure 4: Multiple equilibria in the calibrated model

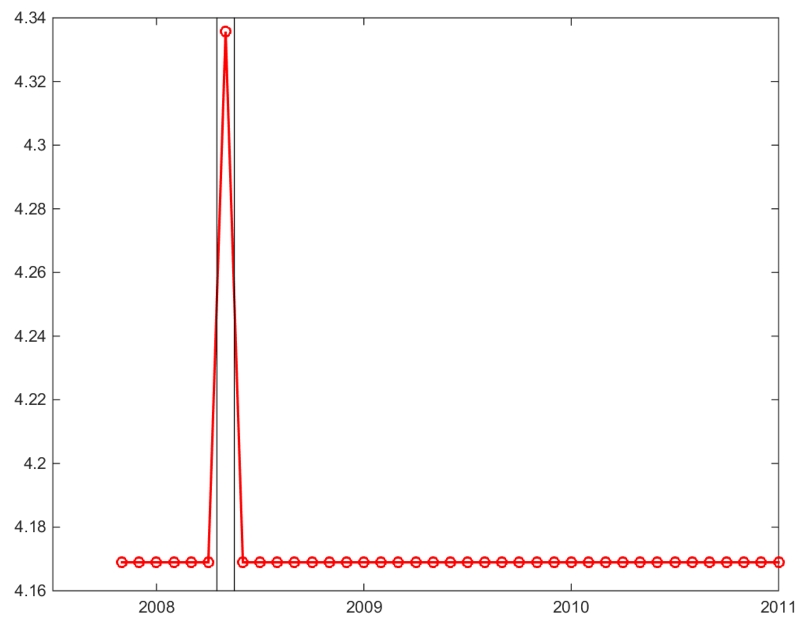

Figure 5: The vacancy costs

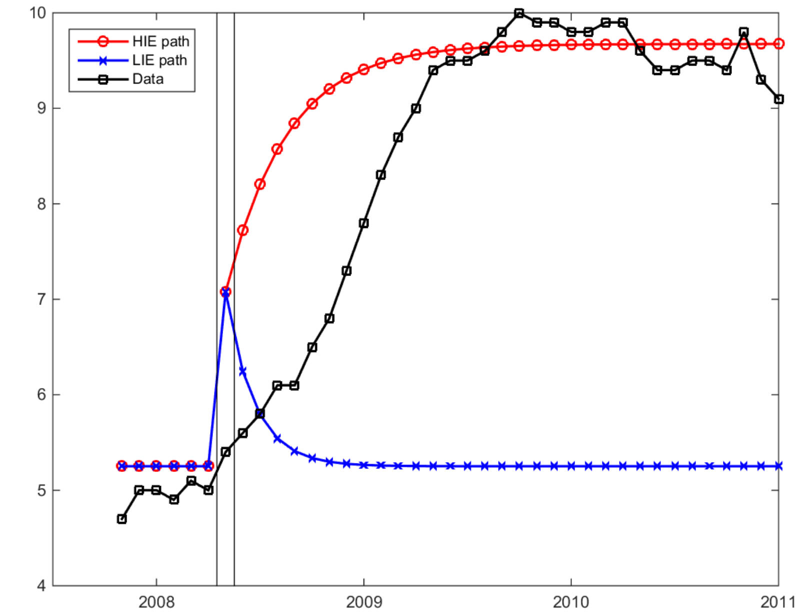

Figure 6: Response to a negative financial shock: unemployment rate, model and data

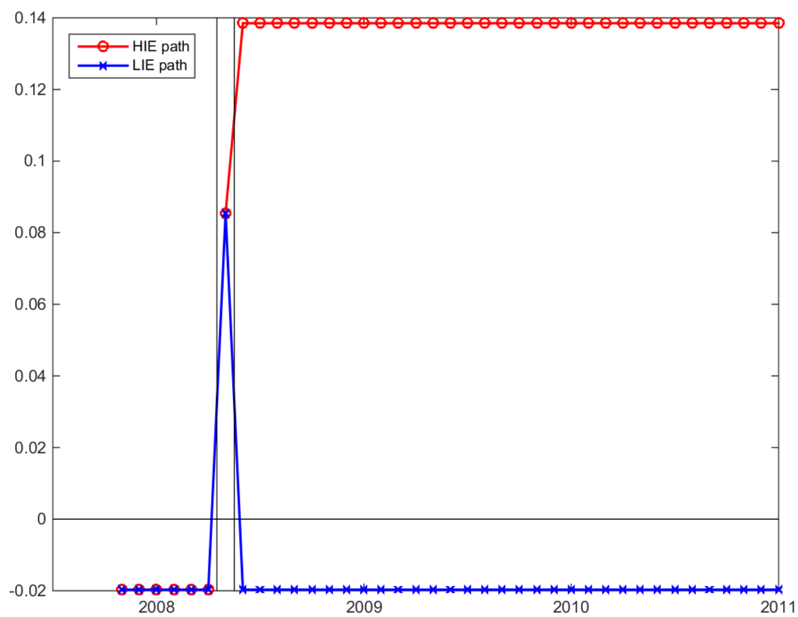

Figure 7: The individual rationality condition

kets. The strategic complementarity makes the multiple equilibria possible in both the static model and the dynamic model. We study the steady state properties of this model and find that searching harder sometimes improves job finding rate, but sometimes it does not. We then calibrate the model to match the features of the U.S. economy, and show that the calibrated model features multiple equilibria. We further show that in response to a transitory financial shock, the model is able to generate a much larger, self-fulfilling and persistent changes in the labor market, which matches well the post-recession high unemployment rate of the U.S. labor market between 2009 and 2011, aka, the jobless recovery.

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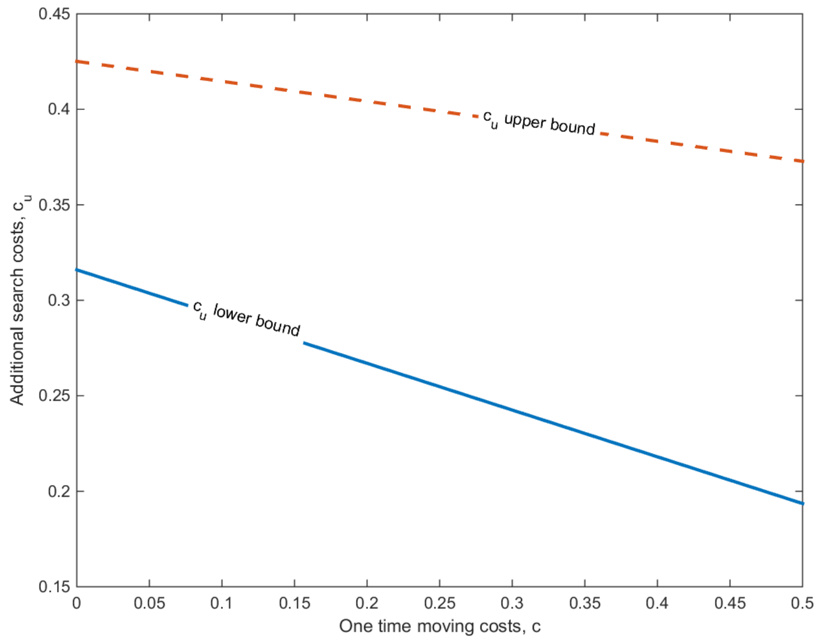

Figure 1: The range of c_u and c

Appendix

A Robustness

The calibrated model features two steady states for a wide range of the worker's additional search costs, c_u, and the one time moving costs, c. For any given $c \in [0, 0.5]$, as long as the value of c_u is between the lower bound and the upper bound, as plotted in Figure 1, the worker's individual rationality constraint holds in both steady states. That is, $\Gamma(\theta) \leq 0$ at the steady state with low search intensity and $\Gamma(\theta) \geq 0$ at the steady state with high search intensity.[18]

[18] Specifically, given c, the lower bound of c_u is derived from $\Gamma_L(\theta_L) \leq 0$, or

$$c_u \geq \beta\theta_L M(\theta_L)\left[1 - \theta_L M(\theta_L)\right]\left[\frac{w - y_u}{\beta\theta_L M(\theta_L) + 1 - \beta(1-\delta)} - c\right].$$

The upper bound of c_u is derived from $\Gamma_H(\theta_H) \geq 0$, or

$$c_u \leq \frac{\beta\theta_H M(\theta_H)\left[1 - \theta_H M(\theta_H)\right]\left\{w - y_u - \left[\beta\theta_H M(\theta_H) + 1 - \beta(1-\delta)\right]c\right\}}{\beta\theta_H M(\theta_H) + 1 - \beta(1-\delta)}$$