

Understanding AH Premium in Chinese Stock Market*

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Preliminary Draft

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Abstract

There are 88 companies (AH share) dual-listed in both China mainland stock markets (A share) and Hong Kong stock market (H share) accounted for 20% of total A share. The ‘Shanghai-Hong Kong Stock Connect’ program starting at November, 2014 makes previously two segmented markets—Shanghai and Hong Kong stock markets—connected. The price difference of AH share in Shanghai and Hong Kong stock markets, measured by Hang Seng China AH Premium Index, instead of converging persistently diverges, and even reaches 50% higher in Shanghai market. We have shown that present-value asset pricing models with heterogeneity agents with different risk aversions or diverse beliefs in the complete market and incomplete markets cannot generate any AH premium. Transaction cost and different dividend taxes between Shanghai and Hong Kong markets also fails to explain such high and volatile AH premium. We, hence, propose an ‘Internal Rationality’ learning model, in which agents don’t know the pricing function from fundamentals to the stock prices and have different subjective beliefs about tomorrow’s capital gains in Shanghai and Hong Kong markets. Our learning model can successfully generate data-like weekly AH premium. We also show that convergence traders with strategy short in Shanghai and long in Hong Kong will lose money with 33% probability.

1 Introduction

This paper studies the stock price difference in the connected Chinese A- and H- share markets, which is an interesting anomaly in asset markets. The stocks listed in Chinese mainland stock exchanges (Shanghai and Shenzhen) are called A-share, and the one listed in Hong Kong exchanges are called H-share. There are 88 companies dual-listed in A-share and H-share markets called AH-share, which are identical with respect to shareholder rights, such as voting and profit-sharing. Most of AH-share companies are big ones, especially state-owned enterprises, accounting for 20% of total market value in A-share market. Hang Seng China AH Premium Index plotted in Figure 1 measures the weighted averaged price difference of AH-share. Index equaling 100 means that A-share are trading at par with H-share, larger than 100 for A shares trading at a premium versus H shares, smaller than 100 for A shares trading at a discount versus H shares.

Figure 1 shows that AH-share prices are always different even though they have the same fundamentals in Shanghai and Hong Kong markets. Before November 2014, Shanghai and Hong Kong markets were segmented that mainland investors are not allowed to invest in Hong Kong market and so for foreign investors in Shanghai market. The price difference of dual-listed stocks in the segmented markets is widely studied in the literature. Fernald and Rogers (2002) attribute the discount of Chinese B-share stock (only for foreigners) to A-share stock (only for the domestic) to the fact that Chinese investors have a higher discount rate than foreigners. Chan, Menkveld and Yang (2008) show the evidence that AB-share premium is caused by the fact that foreign investors, who trade B-shares, have an informational disadvantage relative to domestic investors, who trade A-shares. While, Mei, Scheinkman and Xiong (2009) propose that trading caused by investors' speculative motives can help explain a significant fraction of the price difference between the perfect segmented dual-class AB-shares.

The segmented Shanghai and Hong Kong markets, however, become connected since the starting of Shanghai-Hong Kong stock connect program. The AH premium index should converge to 100 according to the standard present-value asset pricing theory, but it diverges dramatically to arrive at almost 150 and then fluctuates between 120 and 150. There are very few works on the price difference in the connected markets except Froot and Dabora (1999) focusing on only three twin stocks. This paper studies the price difference in the sample with 88 dual-listed stocks.

This paper first investigates whether the present-value heterogeneous asset pricing models can generate sufficiently high, volatile and persistent AH premium. The heterogeneity could be reflected in agents' different discount factors (Fernald and Rogers, 2002), diverse beliefs caused by asymmetric information (Chan, Menkveld and Yang, 2008), and different transaction costs and dividend taxes (Froot and Dabora, 1999). The model environment could be complete market or incomplete market, and stock prices equalling with the discounted sum of expected future dividends makes agents like fundamental investors. We find that different risk aversion, discount factors, and diverse beliefs cannot produce any AH premium, transaction costs are so small that could be ignored, and dividend taxes is possible to generate constant 6% premium. The generalized model we show in section 4 illuminates that prices for A share and H share in the connected markets are the same in each period when we only have variations across agents without variations across two shares.

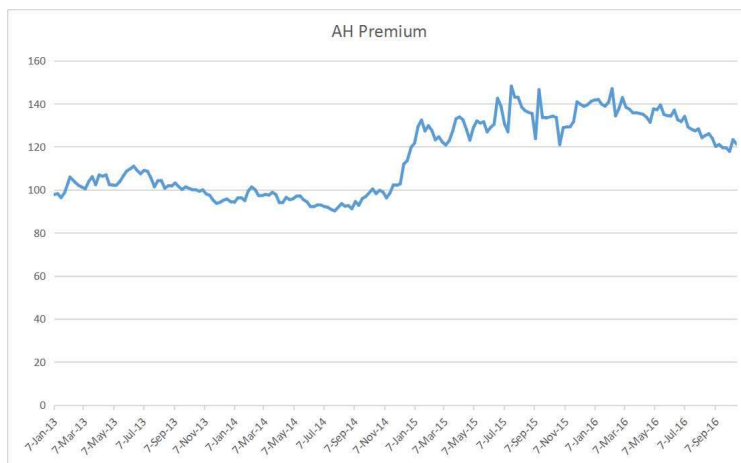


Figure 1: Hang Seng China AH Premium Index

The failure of present-value models in producing AH premium motivates us to propose an 'Internal Rationality' learning model as Adam, Marcet and Nicolini (2016), in which agents who do not know the fundamentals to price mapping and optimize their behaviors based on their subjective beliefs about all variables that are beyond their control. Given the subjective beliefs we specify, agents behave as speculators and optimally update their expectations about capital gains using Kalman filter. Agents' subjective expectations in turn influence equilibrium stock prices, and the realized stock prices feed back into agents' beliefs. If

agents have initial different beliefs or different learning speeds between A-share and H-share, agents can have different subjective beliefs on them which generate different stock prices.

Finally we study the convergence traders' strategy, which relies on the price convergence of similar or identical assets. A typical convergence trade would short sell in AH share in Shanghai market, and long buy it in the Hong Kong market. But the learning model shows that prices cannot converge in the short-run. Since the longest duration of short-selling tool is one-year, we calculate the distribution of money-making for 3, 6, 9 months and one year. We find that convergence traders have a large probability to loss money.

2 Overview of Chinese Stock Market

Chinese stock market is relatively young and started in 1990 with the establishment of two mainland exchanges: the Shanghai Stock Exchange (SSE) and Shenzhen Stock Exchange (SZSE). The number of listed companies was just 13 in the starting time. During the period from 1990 to 2015, the Chinese economy has performed well with averaged annual 10% GDP growth rate. The extraordinary economic growth undoubtedly leads to the rapid growth of financial markets. The market value of Chinese stock market (excluding Hong Kong and Taiwan) reaches \$8.4 trillion at the end of 2015 which makes it the second largest one in the world, even though the ratio of market capitalization to GDP is relatively low at about 60%. The number of listed companies also rises to 2827. The main boards of the Shanghai and Shenzhen Stock Exchanges list larger and more mature stocks, like the NYSE in the US. The Shenzhen Stock Exchange also includes two other boards, the Small and Medium Enterprise Board and the ChiNext Board, also known as the Growth Enterprise Board, which provide capital for smaller and high-technology stocks, like the NASDAQ in the US. Mainland stock market has a dual-share system. Before the starting of Shanghai-Hong Kong Stock connect, mainland investors can invest only in A shares, while foreign investors can invest only in B shares.

Figure 2 shows the dynamics of stock prices indexes in mainland Shanghai and Shenzhen markets from 1995 to 2015 respectively. Mainland stock price experiences two episodes of obvious boom and bust, one is 2006-2007 and the other is 2014-2015. Stock price reached the historical peak in 2007 from the bottom in 2005, then quickly busted. Then, from 2008 to 2014 the market generally

Statistics	SSE	SZSE	S&P500	CRSP
Std.dev. stock return σ_{rs}	17.06%	22.32%	8.17%	8.69%
Std.dev. stock return σ_{PD}	277.83	167.07	47.26	54.94

Table 1: Chinese and US Stock Market Volatility

trended down. Therefore, Allen et al. (2015) thinks that the performance of Chinese stock market has been disappointing, especially compared with the growth of GDP. The market price boomed again in the second half of 2014, and almost doubled in the middle of 2015. One distinguished characteristic in Chinese stock market is that stock trading is new to most of participants, 80% of them are individual investors (Mei, Scheinkman and Xiong, 2009). Given the typical Chinese investor's lack of experience, it is reasonable to hypothesize that these investors would often disagree about stock valuation and as a result would behave more like the speculators. The larger volatility in Chinese stock markets than US shown in table support this.

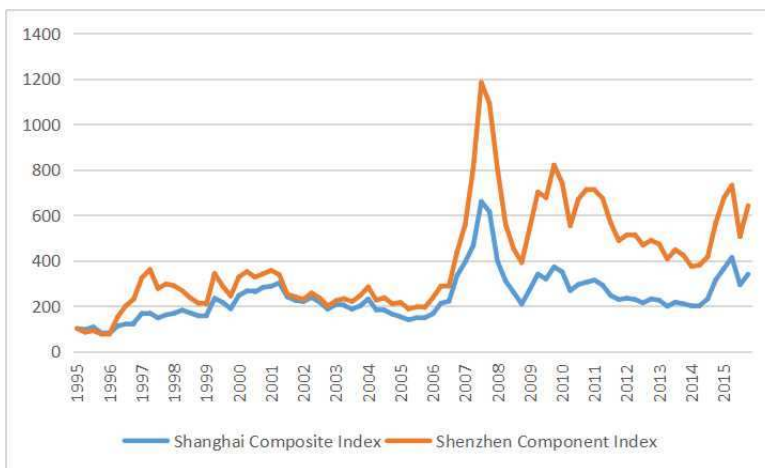


Figure 2: Chinese Stock Price Indexes

3 Present-Value Models

In this section, we build present-value models and explore on the potential factors driving the price difference. We consider variation of discount factor, beliefs on fundamental, dividend tax, transaction fee and relative risk aversion across agents in the complete market and incomplete markets.

3.1 Models in Complete Market

3.1.1 Rational Expectation

Let's describe the economy in complete market. Basically it is a Lucas tree model with two type of agents.

The type i investors in the economy account for a fraction of $\mu^i > 0$ of population $i \in \{1, 2\}$ respectively, where $\mu^1 + \mu^2 = 1$. Type 1 agent stands for mainland investor and 2 for Hong Kong investor. The two types may differ with respect to their degree of risk aversion, discount factor and beliefs on the fundamentals.

In the beginning we endow the agents with right objective beliefs i.e. rational expectation. Investors' portfolio includes A share, H share and contingent bonds. Agents trade A share and H share with each other in this economy. $S_t^{1,A}, S_t^{1,H}, S_t^{2,A}, S_t^{2,H}$ are denoted as A-and H-share stocks that agent 1 and agent 2 buy respectively on period t . One unit of A-share and H-share pay investors the same dividend.

$$D_t^A = D_t^H = D_t$$

For convenience and without loss of generality we assume the exogenous dividend process in the complete market economy is *i.i.d* taking two values of D_h and D_l at each period, where $Prob(D_l) = \pi, Prob(D_h) = 1 - \pi$. We start exploring on complete market with Arrow securities $B_t(D_h)$ and $B_t(D_l)$ that pays 1 unit of consumption if dividend payment on $t + 1$ is high and low respectively. We assume agents hold rational expectation for the dividends process by now. In the following section we will consider what happens if agents have diverse beliefs on fundamentals rather than rational expectation and if the dividends take other standard process.

Commodity goods market clear condition is

$$2D_t = \mu^1 C_t^1 + \mu^2 C_t^2.$$

Arrow securities markets clear conditions are

$$\mu^1 B_t^1(D_j) + \mu^2 B_t^2(D_j) = 0 \quad \forall j = h, l .$$

A and H share market clearing are

$$\mu^1 S_t^{1,Z} + \mu^2 S_t^{2,Z} = 1 \quad \forall Z = A, H.$$

We assume utility function is increasing, concave and continuously differentiable. The investors' maximization problem is

$$\begin{aligned} & \max_{\{C_t^i, S_t^{i,A}, S_t^{i,H}, B_t^i\}} E_0 \sum_{t=0}^{\infty} (\delta^i)^t u_i(C_t^i) \\ \text{s.t.} \quad & S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i + B_t^i(D_h)Q_t(D_h) + B_t^i(D_l)Q_t(D_l) \\ & = S_{t-1}^{i,A}(P_t^A + D_t^A) + S_{t-1}^{i,H}(P_t^H + D_t^H) + B_{t-1}^i \end{aligned}$$

F.O.Cs lead to

$$\delta^1 \frac{u_c^1(C_{t+1}^1)}{u_c^1(C_t^1)} = \delta^2 \frac{u_c^2(C_{t+1}^2)}{u_c^2(C_t^2)} \quad (1)$$

where u_c is marginal utility.

This result of full insurance features complete market. Although agents could have different discount factors and risk aversions, the property of full insurance gives rise to the same stochastic discount factor (*SDF*) as equation (1). Given the stock prices having the present-value expression, the same dividends and *SDF*s, there is no price difference between A-share and H-share. Therefore discount factors and different relative risk aversion across the two agents are not able to drive any price difference in a connected world.

Due to the full insurance property, two types of agents have the same *SDF*s. Here we are exploring the black box that how agents arrive at full insurance through trading contingent bonds. This is not studied in the literature to our best knowledge. For illustration, we fully solve an exercise where both agents have *CRRA* utility with same discount factor but agent 1 is more risk averse than agent 2 by approximating expectations in Euler equations with log linear polynomials.

With state contingent bonds, stock A and stock H are indeterminate 'redundant' assets. So we can keep agents' holding of the two assets fixed over time. Intuitively agent 1 prefers smoother consumption than agent 2 does because agent 1 is more relative risk averse. The full insurance is achieved through agent 1 always buying low contingent bond and selling high contingent bond. We confirm this by numerically solving the quantity of bond holdings. We find that agent 1's consumption are relatively smoother across nature states than agent 2's while their consumption varies with endowment. These observations are shown in Table 2. The algorithm in detail is in appendix A.1.

	c_h	c_l	b_h	b_l
Agent 1	0.8736	0.5514	-0.0036	0.0988
Agent 2	1.1264	0.4486	0.0036	-0.0988

Table 2: Contingent Bond Holding

Different dividend tax and transaction cost are suspected to be eligible in driving the price difference. We then investigate whether this transaction cost and dividends tax could generate AH premium quantitatively enough. Hence we now add tax and cost in the model. Generally, the transaction cost here includes financial tax, exchange rate change and swap charge between RMB and Hong Kong Dollars. $\tau^{1,A}$ and $\tau^{1,H}$ are denoted as dividend tax for mainland investors to trade A share and H share respectively, while $\tau^{2,A}$ and $\tau^{2,H}$ are dividend tax for foreign investors including Hong Kong investors. Similarly, $tc^{i,z} \forall i = 1, 2 \forall z = A, H$ are transaction cost accordingly. It is clear that transaction cost are functions of trading value $P_t^A(S_t^{i,A} - S_{t-1}^{i,A})$ and $P_t^H(S_t^{i,H} - S_{t-1}^{i,H})$. Fortunately the taxes are linear or proportionate in the real world. We use tc to denote the marginal transaction cost. Furthermore in the real world, tc is extremely small relative to dividend tax and AH premium value. We can calibrate the difference of transaction cost, which is only at value -0.64% on the stand of mainland investors and 2.64% on the stand of foreign investors and relatively very small to the difference of tax. Technically transaction cost should be a function of absolute value of trading. Hence given the transaction costs are so small that we neglect the absolute value for the sake of convenience in deducing the derivative. Budget constraint in this case becomes

$$S_t^{i,A}P_t^A + S_t^{i,H}P_t^H + C_t^i + B_t^i(D_h)Q_t(D_h) + B_t^i(D_l)Q_t(D_l) = S_{t-1}^{i,A}(P_t^A + (1 - \tau^{i,A})D_t^A) + S_{t-1}^{i,H}(P_t^H + (1 - \tau^{i,H})D_t^H) + B_{t-1}^i \quad \forall i$$

The agent offering higher price will be the marginal one for the security. In reality $\tau^{1,A}$ is 5%, $\tau^{1,H}$ is 20%, $\tau^{2,A}$ is 10% and $\tau^{2,H}$ is 10% according to the policy term in real world. With these dividend tax value, we have $\frac{1 - \tau^{1,A}}{1 + tc^{1,A}} > \frac{1 - \tau^{2,A}}{1 + tc^{2,A}}$. Hence type 1 is marginal in Shanghai and type 2 is marginal in Hong Kong as can be seen in the F.O.Cs below and type 1 agent (mainland investors) is always the marginal investor with type 2 agent's borrowing constraint binding in A share. And the opposite is true for H share. Without loss of generality, we assume the two agents have log utility to show the role that transaction cost plays.

The F.O.Cs in this case become

$$P_t^A = E_t \delta^1 \frac{C_t^1}{C_{t+1}^1} [P_{t+1}^A + \frac{1 - \tau^{1,A}}{1 + tc^{1,A}} D_{t+1}] \quad 0 \leq S_t^1 \leq \bar{S}$$

$$P_t^A > E_t \delta^2 \frac{C_t^2}{C_{t+1}^2} [P_{t+1}^A + \frac{1 - \tau^{2,A}}{1 + tc^{2,A}} D_{t+1}] \quad S_t^2 = 0$$

Similarly for H share, we have

$$P_t^H > E_t \delta^1 \frac{C_t^1}{C_{t+1}^1} [P_{t+1}^H + \frac{1 - \tau^{1,H}}{1 + tc^{1,H}} D_{t+1}] \quad S_t^1 = 0$$

$$P_t^H = E_t \delta^2 \frac{C_t^2}{C_{t+1}^2} [P_{t+1}^H + \frac{1 - \tau^{2,H}}{1 + tc^{2,H}} D_{t+1}] \quad 0 \leq S_t^2 \leq \bar{S}$$

Hence we obtain

$$P_t^A = E_t \left[\sum_{j=1}^{\infty} (\delta^1)^j \prod_{k=1}^j \frac{C_{t+k-1}^1}{C_{t+k}^1} \frac{1 - \tau^{1,A}}{1 + tc^{1,A}} D_{t+j} \right] \quad (2)$$

$$P_t^H = E_t \left[\sum_{j=1}^{\infty} (\delta^2)^j \prod_{k=1}^j \frac{C_{t+k-1}^2}{C_{t+k}^2} \frac{1 - \tau^{2,H}}{1 + tc^{2,H}} D_{t+j} \right] \quad (3)$$

And since the tax is constant and can be factored out. Dividing (1) by (2) leads to

$$\frac{P_t^A}{P_t^H} = \left(\frac{1 - \tau^{1,A}}{1 - \tau^{2,H}} \right) \left(\frac{1 + tc^{2,H}}{1 + tc^{1,A}} \right)$$

Transaction costs are so small that we ignore them for a while. Hence price ratio is constant over time with approximate ratio of 1.056, which contradicts with the observation that standard deviation of AH premium fluctuates between 1.2 and 1.5. Furthermore it's worthwhile to notice that if $\tau^{1,A} = \tau^{1,H}$ and $\tau^{2,A} = \tau^{2,H}$, then one would notice same price of A and H shares after a quick look at

the F.O.Cs. Hence if mainland investors face the same dividend tax of A share and H share and so do the foreign investors, there would be no price difference in this complete market framework even if dividends taxes are not the same across agents.

3.1.2 Diverse Belief

Furthermore, there is popular narrative in the market that says foreign investors are pessimistic about Chinese economy but Chinese people have optimistic views on the contrary. These diverse beliefs may be due to imperfect information or other reasons. Dealers and market analysts tend to tell this kind of story to rationalize the AH premium. So let's see what happens when two agents have diverse beliefs on fundamental in this environment while set the financial frictions discussed above aside. Towards this end, we depart a bit from rational expectation model in the way that two agents are endowed with diverse beliefs on fundamental.

Let's assume agent 1 is optimistic while agent 2 is pessimistic. More important let's assume agent 1 is relatively right compared to agent 2. We will see that in the complete market agent 1 will take advantage of his information superiority so that he accumulates assets and consume more goods. Formally let $i \in \{o, p\}$ where o stands for optimistic agent and p stands for pessimistic agent. Optimistic agent perceive $Prob(D_h) = u$ while pessimistic agent perceive $Prob(D_l) = v$ where $u > v$. And the true objective probability is again

$$Prob(D_l) = \pi, Prob(D_h) = 1 - \pi$$

Let $\mathbf{1}(D_h)$ and $\mathbf{1}(D_l)$ be the indicator function that take value 1 if D_h and D_l happen respectively. F.O.Cs lead to

$$\begin{aligned} \frac{C_{t+1}^p}{C_{t+1}^o} &= \frac{C_t^p \delta^p v}{C_t^o \delta^o u} \mathbf{1}(D_{t+1}^h) + \frac{C_t^p \delta^p (1-v)}{C_t^o \delta^o (1-u)} \mathbf{1}(D_{t+1}^l) \\ &= \left[\frac{v}{u} \mathbf{1}(D_{t+1}^h) + \frac{1-v}{1-u} \mathbf{1}(D_{t+1}^l) \right] \frac{\delta^p C_t^p}{\delta^o C_t^o} \end{aligned} \quad (4)$$

where $[\frac{v}{u} \mathbf{1}(D_{t+1}^h) + \frac{1-v}{1-u} \mathbf{1}(D_{t+1}^l)]$ is denoted as A_{t+1} for simplicity.

The assumption that agent 1 relatively right leads to $(\frac{v}{u})^\pi (\frac{1-v}{1-u})^{1-\pi} > 1$, which implies that agent 1 will gradually consume the total dividends in the economy consistent with the conclusion in Bloom and Easley (2006).

Rearranging (3) gives

$$\delta^o \frac{C_t^o}{C_{t+1}^o} = A_{t+1} \delta^p \frac{C_t^p}{C_{t+1}^p}$$

which links *SDF* of optimistic agent with that of pessimistic agent.

Then we obtain the pricing function for both A share and H share.

$$\begin{aligned} P_t^{o,A} &= E_t^o \left[\sum_{j=1}^{\infty} (\delta^o)^j \prod_{k=1}^j \frac{C_{t+k-1}^o}{C_{t+k}^o} D_{t+j} \right] \\ &= E_t^o \left[\sum_{j=1}^{\infty} (\delta^o)^j \prod_{k=1}^j \frac{C_{t+k-1}^p}{C_{t+k}^p} A_{t+j} D_{t+j} \right] \\ &= \sum_{j=1}^{\infty} \left[v (\delta^p)^j \prod_{k=1}^j \frac{C_{t+k-1}^p}{C_{t+k}^p} D_{t+j} + (1 - v (\delta^p)^j) \prod_{k=1}^j \frac{C_{t+k-1}^p}{C_{t+k}^p} D_t \right] \\ &= E_t^p \left[\sum_{j=1}^{\infty} (\delta^p)^j \prod_{k=1}^j \frac{C_{t+k-1}^p}{C_{t+k}^p} D_{t+j} \right] \\ &= P_t^{p,A} \end{aligned}$$

Hence both optimistic and pessimistic are marginal investors and they perceive A share exact the same price. Due to the fact that A share and H share release the same amount of dividend at each period, there is no price difference in each period i.e $P_t^{o,A} = P_t^{p,A} = P_t^{o,H} = P_t^{p,H}$. Although the *SDFs* of the two agents are different and linked by A_{t+1} , the subjectively expected *SDFs* are still the same.

Hence the diverse beliefs on dividends can't give rise to price difference.

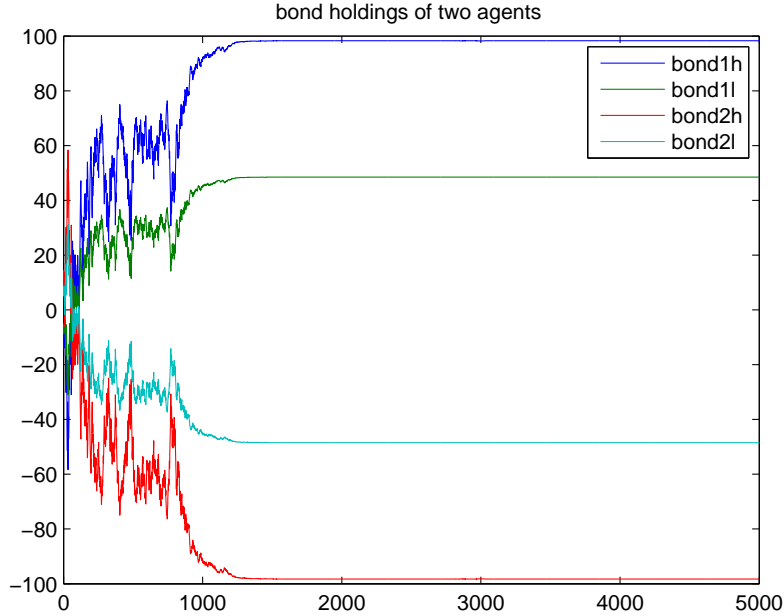


Figure 3: Bond Holding of Two Agents with Arrow Securities

In addition to analytical solution we can also solve this model by approximating expectation in Euler equations with exponentiated log linear polynomial and we find that agents achieve full insurance through contingent bond exchange. The algorithm for this case is on Appendix A.2. Agent 1 is more right with respect to the true probability and so he accumulates assets and consume more while agent 2 accumulates debt and consume less. In the long run agent 1 consumes the total dividend while agent 2 get nothing. This is consistent with Bloom and Easley (2006).

The bond holding converges to steady state value, which is 100 in our setting. The bond holdings are shown on figure 1. However even if the two agents have diverse beliefs about the economy situation, price difference still remains in silence. Hence the diverse belief for the dividend or economy only lead the two agents to hold opposite amount of bonds but not to regard the stock price differently. We also find that it is the relative rightness of the perceived beliefs that drive the more right agent accumulate bonds and the other agent accumulate debt rather than their degree of optimism. For example, even if agent 1 is relatively optimistic, he will also holding bond because of his information advantage. We also try different degree of risk aversion in order to maintain consistency

with rational expectation cases. We find that the results of no price difference remain true and the bond holdings in the long run converge to another steady state level. If we add dividend tax in this context, the price difference occurs but are not quantitatively desirable in the same way that the rational expectation case does.

It is worthwhile to put forward an alternative way to complete the market in this case of simple two value *i.i.d* dividend process. It could be an economy where agents have riskless bond and the two shares. In this case, we will have two contingent budget equations of high and low for three asset. Though the total share holding $S^{i,A} + S^{i,H}$ of each agent is uniquely determined in equilibrium since the rank of return matrix is 2, the individual share holding $S^{i,A}$ and $S^{i,H}$ *per se* are indeterminate. Hence it is necessary to assume for example that agent 1 holds same amount of A share with H share in each time and agent 2 holds the rest shares in the market *i.e.* $S_t^{1,A} = S_t^{1,H}$, $S_t^{2,A} = S_t^{2,H}$. Agents have the exact same consumption plan as they do in the previous case but they now could achieve full insurance by dealing with riskless bonds and stocks .

3.2 Models in Incomplete Market

3.2.1 Rational Expectation

Through out the previous section, the assumption of complete market plays a key role to link the *SDF* of the two agents which enables us to derive analytical price ratio formular. Under the circumstance of incomplete market, agents can't trade Arrow securities freely to adjust stochastic discount factor, which seems to be a problem at the first glance. So we suspect that incomplete market perhaps make some difference. Hence in the following sections, we turn to investigate on incomplete market.

We then consider an enviroment without state contingent bond. The simple discrete dividend process is no longer appropriate for incomplete market. We follows the literature and assume a standard dividend process,

$$\frac{D_t}{D_{t-1}} = a\epsilon_t^d \text{ for } t = 0, 1, 2, \dots$$

where $\log \epsilon_t^d \sim i.i.N(-\frac{s_d^2}{2}, s_d^2)$ and $a \geq 1$.

And budget constraint in this case becomes

$$S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i = S_{t-1}^{i,A} (P_t^A + (1 - \tau^{i,A}) D_t^A) + S_{t-1}^{i,H} (P_t^H + (1 - \tau^{i,H}) D_t^H)$$

$$\forall i = 1, 2$$

To avoid ponzi scheme, the standard borrowing constraint in the literature is assumed,

$$S_t^{i,j} \geq 0, \forall i = 1, 2 \forall j = A, H$$

Typically we don't obtain analytical solutions for price ratio when it comes to incomplete market because we are not equipped with the equation that links the two agents' stochastic discount factors. First order conditions are listed on appendix A.3 to save space. Here to keep parsimonious we assume that two types of agents have the same risk aversions and discount factors, but have different dividend taxes. In the later, we will argue that different risk aversions and discount factors cannot generate any AH premium in the incomplete market. Thus we assume them away here. However dividend taxes have to be kept in this economy.

We solve the model numerically with Parameterized Expectation Approach. We find that during the dynamics, agent 1 holds more and more A shares and less and less H shares while agent 2 does the opposite. This is attributed to the dividend tax structure with $\tau^{1,A} < \tau^{1,H}$ and $\tau^{2,A} = \tau^{2,H}$. However P_t^A and P_t^H are the same during a long period because the four F.O.Cs all hold with equality and the lower bound is reached only after a long time. Hence the two agents are both marginal agent for a long time.

This observation is illustrated in Figure 4. Once the asset holding reaches to the lower bound, as is in the complete market the price difference quantitatively deficient occurs again as a result of dividend tax structure and this case is degenerated to a 'autarky' world in the sense that mainland investors only hold A-share and foreign investors H-share .

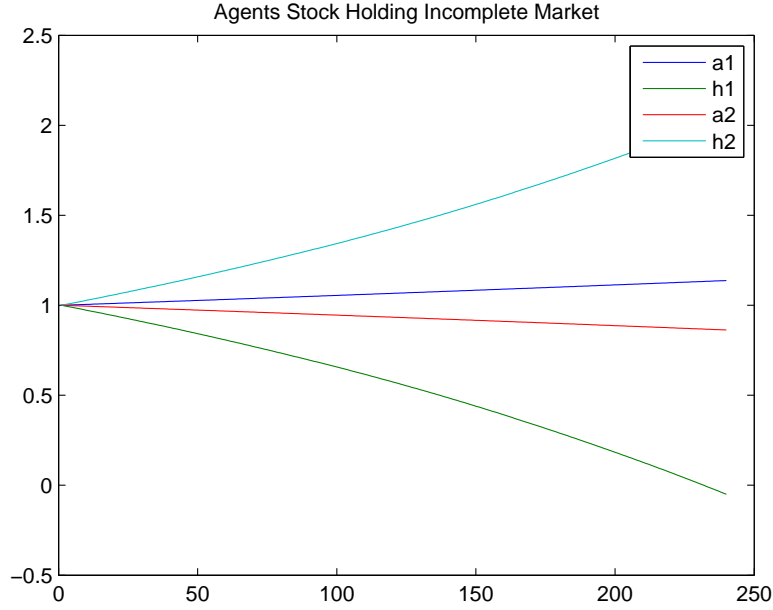


Figure 4: Agents Stock Holding Incomplete Market

3.2.2 Diverse Belief

As in the complete market, we also turn to the case with diverse beliefs. Then agents may have subjective beliefs on dividend process.

$$\frac{D_t}{D_{t-1}} = a_i \epsilon_t^d, \text{ for } i = 1, 2$$

where larger a_i is associated with optimism and smaller a_i with pessimism.

To solve the two agents two assets incomplete market model with PEA method. The PEA algorithm for this case is explained on Appendix A.3. We find that the result in 3.2.1 remains. Hence the diverse belief has nothing to do price difference even in the incomplete market.

4 A General Discussion on Sources of Price Difference under Present-Value Model

We of course have not covered every possible model and it is also impossible to do it. However we give a general discussion here on the criteria determining

whether a given factor has the potential to drive AH premium in the connected market. In this section we'll discuss the necessary conditions to generate price difference in a general way nesting all the cases we have showed in the above analysis and other cases we have not covered yet.

4.1 Variations across Agents

Proposition 1: Variations across agents per se have nothing to do with price difference

This proposition means that prices of the two shares with same dividend stream are same in each period if there are only variations across agents but not variations across two shares. Hence the heterogeneous stuffs across the two agents alone are not able to generate any price difference.

Proof: F.O.Cs without considering borrowing constraint for agent 1 and agent2 are as follows

$$P_t^{1,A} = E_t f(SDF_t^1, P_{t+1}^A, D_{t+1}, z_{1,A}]$$

$$P_t^{2,A} = E_t f(SDF_t^2, P_{t+1}^A, D_{t+1}, z_{2,A}]$$

$$P_t^{1,H} = E_t f(SDF_t^1, P_{t+1}^H, D_{t+1}, z_{1,H}]$$

$$P_t^{2,H} = E_t f(SDF_t^2, P_{t+1}^H, D_{t+1}, z_{2,H}]$$

where f is a generic function and $z_{1,A}$, $z_{2,A}$, $z_{1,H}$ and $z_{2,H}$ are all possible parameters for financial frictions (e.g taxes), all kinds of shocks such as idiosyncratic income shock, and so on associated with agent $i \in \{1, 2\}$ shares $j \in \{A, H\}$ respectively. z could also represent agents' perceived probability for dividends as is shown in the measure change step above, if agents don't have rational expectation. Of course we could rewrite the F.O.Cs highlighting diverse beliefs with subjective belief system in superscript as we have done in the above examples. However it's more directly to regard z as diverse subjective probability on dividends or measure change, which allows us to analyze this case within our framework. $z_{i,j}$ could affect function value by affecting SDF indirectly. SDF could be of any type such as the habit type and the long run risk type as in Campbell and Cochrane (1999) or Bansal and Yaron (2004). For example labor

income shocks could have effect on consumption allocation in equilibrium and thus SDF . Functional form of f should be the same across agents and shares in equilibrium. Furthermore the functional form could varies with different setting so we denote as abstract function f . Therefore f accomodate to every model with variation across agents that we have mentioned or that have not come up with yet. The superscripts on the prices once again just mark the two type of agents and in equilibrium for instance $P_t^{1,A}$ and $P_t^{2,A}$ is the same one price.

When we only have variations across agents, then $z_{1,A} = z_{1,H} = z_1$ and $z_{2,A} = z_{2,H} = z_2$ even though $z_1 \neq z_2$.

For AH-shares on each t , there are two types of cases

(a) Both agents are marginal, then we have

$$P_t^A = E_t f(SDF_t^1, P_{t+1}^A, D_{t+1}, z_{1,A}) \quad 0 \leq S_t^1 \leq \bar{S}$$

$$P_t^A = E_t f(SDF_t^2, P_{t+1}^A, D_{t+1}, z_{2,A}) \quad 0 \leq S_t^2 \leq \bar{S}$$

Because $z_{1,H} = z_{1,S}$ and $z_{2,H} = z_{2,S}$, then the above F.O.Cs also apply to H share. i.e

$$P_t^H = E_t f(SDF_t^1, P_{t+1}^H, D_{t+1}, z_{1,A}) \quad 0 \leq S_t^1 \leq \bar{S}$$

$$P_t^H = E_t f(SDF_t^2, P_{t+1}^H, D_{t+1}, z_{2,A}) \quad 0 \leq S_t^2 \leq \bar{S}$$

Then A share and H share entertain exact the same F.O.Cs in this case and hence the price, though z_1 and z_2 could be different. Both agent 1 and agent 2 are marginal for the two shares.

(b) One type of agent is marginal and the other is not. Without loss of generality let's assume type 1 agent is marginal then we have

$$P_t^A = E_t f(SDF_t^1, P_{t+1}^A, D_{t+1}, z_{1,A}) \quad 0 \leq S_t^1 \leq \bar{S}$$

$$P_t^A > E_t f(SDF_t^2, P_{t+1}^A, D_{t+1}, z_{2,A}) \quad S_t^2 = 0$$

Same argument with (a). When $z_{1,H} = z_{1,A}$ and $z_{2,H} = z_{2,A}$, then the above F.O.Cs also apply to H share. That is

$$P_t^H = E_t f(SDF_t^1, P_{t+1}^H, D_{t+1}, z_{1,H}) \quad 0 \leq S_t^1 \leq \bar{S}$$

$$P_t^A > E_t f(SDF_t^2, P_{t+1}^H, D_{t+1}, z_{2,H}] \quad S_t^2 = 0$$

Then A share and H share entertain exact the same F.O.Cs in this case. We observe that if agent 1 is marginal for A share, then he must be for H share.

By the observation from the above two cases, the marginal investor is of the same type for both shares in each period, whether both two agents are marginal or just one type is marginal, though the marginal type of agent could vary over time.

We use m_t to denote the marginal agent pricing the asset in period t in equilibrium:

$$m_t = \arg \max_{i \in \{1,2\}} E_t f(SDF_t^i, P_{t+1}, D_{t+1}, z_i)$$

Then we find

$$P_t^A = E_t f(SDF_t^{m_t}, P_{t+1}^A, D_{t+1}, z_{m_t})$$

$$P_t^H = E_t f(SDF_t^{m_t}, P_{t+1}^H, D_{t+1}, z_{m_t}) \quad \forall t$$

In order to make the above equations hold for every period, we have to make $P_t^A = P_t^H$. We can also see it by doing one more step.

By mapping dividends to stock price and assuming no bubbles we obtain

$$\begin{aligned} P_t^A &= E_t g(\{SDF_{t+j-1}^{m_{t+j-1}}\}_{j=1}^{\infty}, \{D_{t+j}\}_{j=1}^{\infty}, \{z_{m_{t+j-1}}\}_{j=1}^{\infty}) \\ P_t^H &= E_t g(\{SDF_{t+j-1}^{m_{t+j-1}}\}_{j=1}^{\infty}, \{D_{t+j}\}_{j=1}^{\infty}, \{z_{m_{t+j-1}}\}_{j=1}^{\infty}) \quad \forall t \end{aligned}$$

where g is a new function deduced by recursive summation associated with function f .

For the standard function f linear in P , we will have

$$\begin{aligned} P_t^A &= \sum_{j=1}^{\infty} E_t g'(\{SDF_{t+j-1}^{m_{t+j-1}}\}_{j=1}^{\infty}, \{z_{m_{t+j-1}}\}_{j=1}^{\infty}, D_{t+j}) \\ P_t^H &= \sum_{j=1}^{\infty} E_t g'(\{SDF_{t+j-1}^{m_{t+j-1}}\}_{j=1}^{\infty}, \{z_{m_{t+j-1}}\}_{j=1}^{\infty}, D_{t+j}) \quad \forall t \end{aligned}$$

where g' is the function associated with the standard form of function f .

Hence prices for A share and H share are the same for each period when we only have variations across agents without variations across two shares. Intuitively if there is nothing different across shares, they are the same goods. Then no matter how prices are determined in the present-value model equilibrium, there should not be any price difference.

Thus diverse belief, different discount factors, different incomes among the two agents do not give rise to the price difference. All these stories are about variation across agent. Hence they can't generate any AH premium.

4.2 Variations across Shares could lead to Price Difference

We have to make a given agent regard the A-share and H-share as difference shares rather than make something different across the two agents, since connect enable the agents to allocate their wealth among A-share and H-share. It's no longer the old case of segmented market where all the difference parameters and variables across the two agents could drive price difference and each agent is marginal for the share market he can deal with in equilibrium. Therefore the variation across shares for a agent rather than across agents matters under the environment of rational expectation, it must be that there is something different between A-share and H-share for at least one agent. To put it another way, A shares should benefit one agent more than the other shares do.

However it's very not easy to make an agent perceive A share and H share differently, since they give the same dividends. Transaction cost is such an example. But it can't induce the desirable results.

Some people hold the long-standing view that Chinese government directly control Shanghai stock market frequently. But this is not true. Since 2000 it only happens one time. When Shanghai stock price bubble burst on the end of June 2015, Chinese government required state-owned investment banks to support stock price by taking long positions to avoid severe financial crisis in the worry of the high leverage hold by many Chinese investors. When stock prices was stabilized in August, Chinese governments intervention quickly stepped away.

Another point is about liquidity. Since mainland China has strict control on capital inow, someone perhaps argue that stocks in Shanghai market can provide additional liquidity value for foreigners. For example, people may think that foreigners can invest in mainland real estate market with Chinese currency RMB

they get after they close their positions in AH account. But Chinese government imposes many restrictions for foreigners buying house in China such as duration of stay in China and the proof of income sources, etc. And more crucial point is that foreign investors and Hong Kong investors are required to be paid with US dollars after they clear their positions rather than RMB. Hence there are no such additional benefit investing in Shanghai.

We want to provide a parsimonious way to understand this AH premium. When agents don't know the pricing mapping from fundamental to stock price and behave like speculators. One agent's different belief on price growth of A share and H share could make the agent perceive A share and H share not the same stock, which matches banker, trader and normal Chinese people's view on the stock market. We are not claiming that we know exactly what are going on in their mind but this sort of story is a dominant view in Chinese market. Hence in the following section we turn to a parsimonious learning model.

5 An 'Internal Rationality' Learning Model

Section 3 and 4 have shown that heterogeneous agents asset pricing models with diverse beliefs, different transaction costs and dividend taxes in both complete and incomplete markets are not able to generate sufficient AH premium. This section, hence, presents an 'Internal Rationality' learning model based on Adam, Marcet and Nicolini (2016) to explain such high and volatile AH premium.

5.1 Model Environment

A unit of AH share stock with dividend claim D_t can be traded in both Shanghai and Hong Kong markets. In addition to D_t , each agent receives an endowment Y_t of perishable consumption goods. So the total supply of the consumption goods in the economy is then given by the feasibility condition $C_t = Y_t + 2D_t$. Following traditional setting in asset pricing literature, dividend and endowment growth rates follow i.i.d. lognormal processes

$$\frac{D_t}{D_{t-1}} = a\epsilon_t^d, \log \epsilon_t^d \sim iiN\left(-\frac{s_d^2}{2}, s_d^2\right) \quad (5)$$

$$\frac{C_t}{C_{t-1}} = a\epsilon_t^c, \log \epsilon_t^c \sim iiN\left(-\frac{s_c^2}{2}, s_c^2\right) \quad (6)$$

where endowment and dividend growth rates share the same mean a , and $(\log \epsilon_t^d, \log \epsilon_t^y)$ are joint-normally distributed with correlation $\rho_{y,d}$, and s_d and s_c are standard deviations of this joint normal distribution.

The economy is populated by a unit mass of infinite-horizon agents. We model each agent $i \in [0, 1]$ to have the same standard time-separable CRRA utility function and the same subjective beliefs. This fact, however, is not the common knowledge among agents.

The specification of agent i 's expected life-time utility function is

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \quad (7)$$

where C_t^i is the consumption profile of agent i , δ denotes the time discount factor, and γ is the risk-aversion parameter. Instead of the objective probability measure, expectation is formed using the subjective probability measure \mathcal{P} that describes probability distributions for all external variables. Section 5.2 contains more details.

Agent's choices are subjected to standard budget constraint as following

$$C_t^i + P_t^A S_t^{A,i} + P_t^H S_t^{H,i} = (P_t^A + D_t) S_{t-1}^{A,i} + (P_t^H + D_t) S_{t-1}^{H,i} + Y_t \quad (8)$$

where $S_t^{A,i}$ and $S_t^{H,i}$ are the new units of A share and H share stock agent i buys in period t , P_t^A and P_t^H are the prices of A share and H share. To avoid Ponzi schemes and to insure existence of a maximum the following bounds are assumed to hold

$$\begin{aligned} \underline{S} &\leq S_t^{A,i} \leq \bar{S} \\ \underline{S} &\leq S_t^{H,i} \leq \bar{S} \end{aligned}$$

We only assume the bounds \underline{S} and \bar{S} are finite.

5.2 Probability Space

This subsection explicitly describes the general joint probability space of the external variables. In the competitive economy, each agent considers the joint process of endowment, dividend and stock prices $\{Y_t, D_t, P_t^A, P_t^H\}$ as exoge-

nous to his decision problem. Rational expectations imply that agents exactly know the mapping from a history of endowment Y_t and dividend D_t to equilibrium stock price P_t^A and P_t^H . Stock prices hence just carry redundant information. But if the rational expectation assumption is relaxed, as shown in Adam and Marcat (2011) such that agents do not know such mapping because of the non-existence of common knowledge on agents' identical preferences and beliefs, then equilibrium stock price P_t^A and P_t^H should be included in the underlying state space. We then define the probability space as $(\mathcal{P}, \mathcal{B}, \Omega)$ with \mathcal{B} denoting the corresponding σ -Algebra of Borel subsets of Ω and \mathcal{P} denoting the agent's subjective probability measure over (\mathcal{B}, Ω) . The state space Ω of realized exogenous variables is

$$\Omega = \Omega_Y \times \Omega_D \times \Omega_{P^A} \times \Omega_{P^H}$$

where Ω_X is the space of all possible infinite sequences for the variable $X \in [Y, D, P^A, P^H]$. Thereby, a specific element in the set Ω is an infinite sequence $\omega = \{Y_t, D_t, P_t^A, P_t^H\}_{t=0}^\infty$. The expected utility with probability measure \mathcal{P} is defined as

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \equiv \int_{\omega} \sum_{t=0}^{\infty} \delta^t \frac{C_t^i(\omega)^{1-\gamma}}{1-\gamma} d\mathcal{P}(\omega) \quad (9)$$

Agent i makes contingent plans for endogenous variables $C_t^i, S_t^{A,i}, S_t^{H,i}$ according to the policy function mapping in the following

$$(C_t^i, S_t^{A,i}, S_t^{H,i}) : \Omega^t \rightarrow R^3$$

where Ω^t represents the set of histories from period zero up to period t .

5.3 Optimality Conditions

Since the objective function is concave and the feasible set is convex, the agent's optimal plan is characterized by the first order conditions

$$(C_t^i)^{-\gamma} P_t^A = \delta E_t^{\mathcal{P}} ((C_{t+1}^i)^{-\gamma} (P_{t+1}^A + D_{t+1})) \quad (10)$$

$$(C_t^i)^{-\gamma} P_t^H = \delta E_t^{\mathcal{P}} ((C_{t+1}^i)^{-\gamma} (P_{t+1}^H + D_{t+1})) \quad (11)$$

Before explore why subjective beliefs can explain AH premium, we first briefly review the unique RE solution given by

$$P_t^{A.RE} = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta a^{1-\gamma} \rho_\epsilon} D_t \quad (12)$$

$$P_t^{H.RE} = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta a^{1-\gamma} \rho_\epsilon} D_t \quad (13)$$

where $\rho_\epsilon = E_t^{\mathcal{P}}[(\epsilon_{t+1}^c)^{1-\gamma} \epsilon_{t+1}^d] = e^{\gamma(1+\gamma)\frac{s_c^2}{2}} e^{-\gamma\rho_{c,d} s_c s_d}$. Obviously, RE solution always generates $P_t^{A.RE} = P_t^{H.RE}$.

We now characterize the equilibrium outcome under learning. According to the arguments in Adam, Marcet and Nicolini (2016), absent from strict rational expectations we may have $E_t^{\mathcal{P}}[C_{t+1}^i] \neq E_t^{\mathcal{P}}[C_{t+1}]$ even if in the equilibrium $C_t^i = C_t$ holds ex-post. But we can make the same approximations in the following as they do

$$E_t^{\mathcal{P}}\left[\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\gamma}(P_{t+1}^A + D_{t+1})\right] \simeq E_t^{\mathcal{P}}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(P_{t+1}^A + D_{t+1})\right] \quad (14)$$

$$E_t^{\mathcal{P}}\left[\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\gamma}(P_{t+1}^H + D_{t+1})\right] \simeq E_t^{\mathcal{P}}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(P_{t+1}^H + D_{t+1})\right] \quad (15)$$

The following assumption provides sufficient conditions for this to be the case:

Assumption 1 We assume that Y_t is sufficiently large and the $E_t^{\mathcal{P}} P_{t+1}^{A(H)} / D_t < \bar{M}$ for some $\bar{M} < \infty$ so that, given finite asset bounds \underline{S} and \bar{S} , the approximations (14) and (15) hold with sufficient accuracy.

We then can define the subjective expectations of risk-adjusted stock price growths as

$$\beta_t^A \equiv E_t^{\mathcal{P}}\left[\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\gamma}(P_{t+1}^A)\right] \quad (16)$$

$$\beta_t^H \equiv E_t^{\mathcal{P}}\left[\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\gamma}(P_{t+1}^H)\right] \quad (17)$$

We also assume that agents know the true processes of consumption and dividend growths. The definitions of β_t^A and β_t^H together with two first order conditions give rise to the asset pricing equations

$$P_t^A = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta \beta_t^A} D_t \quad (18)$$

$$P_t^H = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta \beta_t^H} D_t \quad (19)$$

From equation (18) and (19), our learning model is possible to generate AH premium if $\beta_t^A \neq \beta_t^H$.

5.4 Beliefs Updating Rule

This section fully specifies the subjective probability distribution \mathcal{P} , and derives the optimal belief updating rule for subjective beliefs β_t^A and β_t^H . Similar to the arguments in Adam, Marcet and Nicolini (2016), in agents' beliefs the true processes for risk-adjusted stock price growths in both Shanghai and Hong Kong markets can be modeled as the sum of a persistent component and of a transitory component

$$\begin{aligned} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{t+1}^A}{P_t^A} &= e_t^A + \epsilon_t^A, \quad \epsilon_t^A \sim iiN(0, \sigma_{\epsilon, A}^2) \\ e_t^A &= e_{t-1}^A + \xi_t^A, \quad \xi_t^A \sim iiN(0, \sigma_{\xi, A}^2) \end{aligned} \quad (20)$$

$$\begin{aligned} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{t+1}^H}{P_t^H} &= e_t^H + \epsilon_t^H, \quad \epsilon_t^H \sim iiN(0, \sigma_{\epsilon, A}^2) \\ e_t^H &= e_{t-1}^H + \xi_t^H, \quad \xi_t^H \sim iiN(0, \sigma_{\xi, A}^2) \end{aligned} \quad (21)$$

where e_t^A and e_t^H are persistent components, ϵ_t^A and ϵ_t^H are transitory components. One way to justify these processes is that they are compatible with RE. According to equation (12) and (13), the rational expectation of risk-adjusted price growth is $E_t[(\frac{C_{t+1}}{C_t})^{-\gamma} \frac{P_{t+1}^A}{P_t^A}] = E_t[(\frac{C_{t+1}}{C_t})^{-\gamma} \frac{P_{t+1}^H}{P_t^H}] = a^{1-\gamma} \rho_\epsilon$. Hence, the previous setup encompasses the rational expectation equilibrium as a special case when agents believe $\sigma_{\xi, A}^2 = \sigma_{\xi, H}^2 = 0$ and assign probability one to $e_0^A = e_0^H = a^{1-\gamma} \rho_\epsilon$.

Then, we allow for a non-zero variance $\sigma_{\xi, A}^2$ and $\sigma_{\xi, H}^2$. Agents can only observe the realizations of risk-adjusted growths (the sum of persistent and transitory components), hence the requirement to forecast the persistent components e_t^A and e_t^H calls for a filtering problem. The priors of agents' beliefs can be centered at their rational expectation values and given by

$$e_0^A \sim N(a^{1-\gamma} \rho_\epsilon, \sigma_{0, A}^2)$$

$$e_0^H \sim N(a^{1-\gamma} \rho_\epsilon, \sigma_{0, H}^2)$$

and the variances of prior distributions should be set up to equal with steady

state Kalman filter uncertainty about e_t^A and e_t^H

$$\sigma_{0,A}^2 = \frac{-\sigma_{\xi,A}^2 + \sqrt{\sigma_{\xi,A}^4 + 4\sigma_{\xi,A}^2\sigma_{\epsilon,A}^2}}{2}$$

$$\sigma_{0,H}^2 = \frac{-\sigma_{\xi,H}^2 + \sqrt{\sigma_{\xi,H}^4 + 4\sigma_{\xi,H}^2\sigma_{\epsilon,H}^2}}{2}$$

Then agents' posterior beliefs will be

$$e_t^A \sim N(\beta_t^A, \sigma_{0,A}^2)$$

$$e_t^H \sim N(\beta_t^H, \sigma_{0,H}^2)$$

And the optimal updating rule implies that the evolution of β_t^A and β_t^H is taking the form just as constant gain adaptive learning

$$\beta_t^A = \beta_{t-1}^A + \frac{1}{\alpha^A} \left(\left(\frac{C_{t-1}}{C_{t-2}} \right)^{-\gamma} \frac{P_{t-1}^A}{P_{t-2}^A} - \beta_{t-1}^A \right) \quad (22)$$

$$\beta_t^H = \beta_{t-1}^H + \frac{1}{\alpha^H} \left(\left(\frac{C_{t-1}}{C_{t-2}} \right)^{-\gamma} \frac{P_{t-1}^H}{P_{t-2}^H} - \beta_{t-1}^H \right) \quad (23)$$

where $\alpha^A = \frac{\sigma_{\xi,A}^2 + \sqrt{\sigma_{\xi,A}^4 + 4\sigma_{\xi,A}^2\sigma_{\epsilon,A}^2}}{2\sigma_{\xi,A}^2}$ and $\alpha^H = \frac{\sigma_{\xi,H}^2 + \sqrt{\sigma_{\xi,H}^4 + 4\sigma_{\xi,H}^2\sigma_{\epsilon,H}^2}}{2\sigma_{\xi,H}^2}$ given by optimal (Kalman) gain.

The adaptive learning schemes as equation (22) and (23) as well as pricing equation (18) and (19) can generate a high stock markets volatility coming from the feedback channel between stock price $P_t^{A(H)}$ and subjective beliefs $\beta_t^{A(H)}$. According to equation (18) or (19), a high (low) $\beta_t^{A(H)}$ will lead to a high (low) realized stock price. This will reinforce the subjective beliefs to induce a even higher (lower) $\beta_{t+1}^{A(H)}$ through equation (22) or (23) leading to much higher (lower) stock price so on. The self-referential aspect of the model is the key for producing stock market volatility. Therefore, a difference of initial beliefs between β^A and β^H or in learning speeds α^A and α^H is promising to generate persistently different prices between A share and H share.

Finally, in order to avoid the explosion of stock price $P_t^{A(H)}$ agents' subjective belief $\beta_t^{A(H)}$ is replaced by $\omega(\beta_t^{A(H)})$, the projection facilities. ¹

¹We present the details of projection facilities in Appendix A.5.

Parameters	Value
γ	5
$\sigma_{\Delta D/D}$	0.0204
$\sigma_{\Delta C/C}$	0.0025
a	1.0014
$\rho_{c,d}$	-0.03
δ	0.999
$1/\alpha^A$	0.0030
$1/\alpha^H$	0.0015

Table 3: Parameters Values for Learning Model

5.5 Quantitative Performance

This subsection presents the simulation outcomes of our learning model. We simulate our model at weekly frequency. We first give value to the coefficient of relative risk-aversion γ at 5 following Adam, Marcet and Nicolini (2016), then calibrate the mean and standard deviation of dividend growth a , $\sigma_{\Delta D/D}$, the standard deviation of consumption growth $\sigma_{\Delta C/C}$, the correlation between consumption growth and dividend growth $\rho_{c,d}$ using Shanghai stock market data and Chinese consumption per capita data. We also calibrate δ to match annual 4% interest rate. Meanwhile, we give values to α^A and α^H such that $\alpha^A < \alpha^H$, which can come from agents' subjective beliefs that $\sqrt{\frac{1}{4\sigma_{\xi,A}^2} + \frac{\sigma_{\xi,A}^2}{\sigma_{\xi,A}^2}} < \sqrt{\frac{1}{4\sigma_{\xi,H}^2} + \frac{\sigma_{\xi,H}^2}{\sigma_{\xi,H}^2}}$. This is not arbitrary setting because the realized data of P_t^A and P_t^H can support this inequality if we use MLE method to estimate related parameters given the data follow the processes (20) and (21). Table 3 contains the parameter values.

We simulate the learning model for 100 periods to match almost 2 years' sample period since November 2014. Figure 5 presents the simulated A share price P_t^A and H share price P_t^H , and Figure 6 presents the simulated AH premium. We set the $\beta_1^A = \beta_t^H$ and β_2^A a little larger than β_2^H , which are consistent with data observations. Then, a higher learning speed in A share leads P_t^A to fluctuate more strongly than P_t^H even if two prices dynamics keep the similar shape. Comparing with figure, the model simulated prices trajectory close to data. More importantly, the shape of simulated AH premium captures several important factors of data: 1. starting from somewhere around 100, 2. persistently increasing to about 150, and 3. decreasing to about 120 after 2 years. Hence, our learning does a much better job in generating data-like AH premium

than the models in section 3.

6 Convergence Traders' Strategy

A typical convergence trader is to bet that price difference between two assets with identical, or similar fundamentals will narrow in the future. The convergence trade would hold long positions in one asset he considers undervalues and short positions in the other asset considered to be overvalued. A famous example is that the hedge fund Long-Term Capital Management (LTCM) expected the convergence of bond yields in the emerging market countries and US (Edwards, 1999). They bought emerging markets' bonds and sold short US government bonds. The spread of bond yields, however, widened because of the deterioration of Asian financial crisis and the default of Russian Sovereign debt. The unexpected widening leads to the near-collapse of LTCM. Besides the case of LTCM, Wall Street Journal reported in June 2015 that many convergence traders participating in AH share market by short selling A share and long buying H share, but encountered a huge lost at the early stage after AH connects.

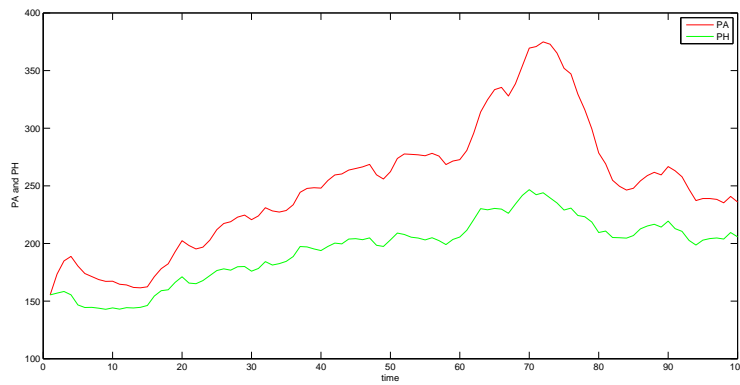


Figure 5: Simulated Stock Prices of A Share and H Share

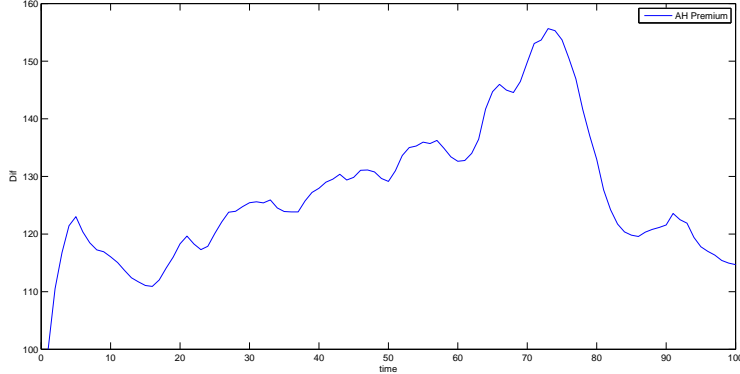


Figure 6: Simulated AH Premium 14

Xiong (2001) studies the convergence trading strategy in the model which has three types of traders: noise traders, convergence traders and long-term traders. He finds that convergence traders reduce asset price volatility in general. But when an unfavorable shock causes them to suffer substantial capital losses, they liquidate their positions, thereby amplifying the original shock. This section considers the convergence traders in another way that they take the stock prices set by learning agents as given as if the learning model above is the true model, and studies the probability distribution of profits when they hold convergence trader strategy.

The convergence traders at period 100 expect that AH premium should narrow in the future, hence they short sell 1 unit of A share stock and use the money from selling to buy H share stock. To implement short selling in Chinese stock market, convergence traders should have as much money as 50% of short selling value in their account as security deposit. In every period, the guarantee ratio gr_t should be calculated as

$$gr_t = \frac{0.5 * P_{100}^A + P_t^H * \frac{P_{100}^A}{P_{100}^H}}{P_t^A}$$

If $gr_t < 130\%$, convergence traders will be asked to add more security deposit to avoid forced liquidation. The maximum duration of short selling is 1 year. We now Monte-Carlo simulate the learning model 10,000 paths with each path representing from 100th period to 152th period. The forced liquidation probability of $gr_t < 130\%$ in a period can reach as high as 13%. We can also calculate the

m_t	Mean	Std	$\Pr(m_t < 0)$
3m	0.817	10.657	0.384
6m	0.662	17.345	0.365
9m	0.713	23.231	0.338
1y	0.887	26.659	0.323

Table 4: The Distribution of Profits from Convergence Trading Strategy

distribution of profits $m_t = P_t^H * \frac{P_{100}^A}{P_{100}^H} - P_t^A$ when $t = 113, 126, 139$ or 152 corresponding to 3 months, 6 months, 9 months and 1 year. Table 4 shows the results. We find the mean of m_t much smaller than the standard deviation of it and a large probability of losing money. Different from Xiong (2001), our learning model cannot guarantee the convergence of AH premium. Therefore, it is not surprising that convergence traders have large probability to lose money.

7 Conclusion

This paper studies the AH premium, which is an interesting anomaly in asset markets.

We have shown that asset pricing models with heterogeneity agents with different risk aversions or diverse beliefs in the complete market and incomplete markets cannot generate any AH premium. Transaction cost and different dividend taxes between Shanghai and Hong Kong markets also fails to explain such high and volatile AH premium. We propose an ‘Internal Rationality’ learning model, in which agents don’t know the pricing function from fundamentals to the stock prices and have different subjective beliefs about tomorrow’s capital gains in Shanghai and Hong Kong markets. Our learning model can successfully generate data-like weekly AH premium. Finally we show that convergence traders with strategy short in Shanghai and long in Hong Kong will lose money with 33% probability.

This maybe an evidence that Chinese investors are more speculative, which seems to be related to the higher stock price volatility in China than that in U.S. and the fact that stock price is highly negative correlated with PMI index for economy prospect in China during the year 2015. These topics worth to be explored in the future research.

References

- [1] Klaus Adam, Johannes Beutel, Albert Marcet, and Sebastian Merkel. Can a financial transaction tax prevent stock price booms? *Journal of Monetary Economics*, 76:S90–S109, 2015.
- [2] Klaus Adam, Albert Marcet, and Juan Pablo Nicolini. Stock market volatility and learning. *The Journal of Finance*, 71(1):33–82, 2016.
- [3] Franklin Allen, Jun Qian, Susan Chenyu Shan, and Julie Lei Zhu. Explaining the disconnection between China’s economic growth and stock market performance. *China International Conference in Finance*, pages 9–12, 2015.
- [4] Ravi Bansal and Amir Yaron. Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance*, 59(4):1481–1509, 2004.
- [5] Lawrence Blume and David Easley. If you’re so smart, why aren’t you rich? belief selection in complete and incomplete markets. *Econometrica*, 74(4):929–966, 2006.
- [6] John Y Campbell and John H Cochrane. By force of habit: A consumption-based explanation of aggregate stock market behavior. 1995.
- [7] Kalok Chan, Albert J Menkveld, and Zhishu Yang. Information asymmetry and asset prices: Evidence from the China foreign share discount. *The Journal of Finance*, 63(1):159–196, 2008.
- [8] John Fernald and John H Rogers. Puzzles in the Chinese stock market. *Review of Economics and Statistics*, 84(3):416–432, 2002.
- [9] Kenneth A Froot and Emil M Dabora. How are stock prices affected by the location of trade? *Journal of Financial economics*, 53(2):189–216, 1999.
- [10] Jianping Mei, Jose Scheinkman, and Wei Xiong. Speculative trading and stock prices: Evidence from Chinese AB share premia. *National Bureau of Economic Research*, 2005.
- [11] Hu Zhanghong and Wang Xiaokun. Empirical analysis of AH premium. *Chinese Economic Research Journal*, 4:119–131, 2008.

Appendix

A.1 Algorithm for two agents two shares with rational expectation in complete market

Step 1: Simulate $\{D_t\}$ for a long time. Solve for $\frac{u'(C_0^1)}{u'(C_0^2)}$ by simulating the economy especially $\{\{C_t^{1,n}, C_t^{2,n}\}_{t=0}^T\}_{n=1}^N$ given initial bond holding B_{-1} , since we have one equation of present value budget constraint for B_{-1} and one unknown. Hence we got the equilibrium λ . It could be solved by iterating on λ or for example just use fsolve. The equation is as follows:

$$\frac{1}{N} \sum_{n=1}^N \sum_{t=0}^T \delta^{1,t} \frac{u'(C_t^1)}{u'(C_0^1)} (C_{t+j}^1 - D_{t+j}) = B_{-1}$$

Here we simulate N times for T period economy. T could be small for example 100.

Step 2: Find $\{C_t^1, C_t^2\}_{t=0}^T$ by simulating a very long sequence of D

ie. At time t , given $\frac{u'(C_t^1)}{u'(C_t^2)} = \lambda$ and market clear condition, C_t^1 and C_t^2 can be solved .

Here we are facing a convex problem. Thus theoretically we should get unique solution though the two conditions lead to a polynomial of C .

Step 3: Solve for the realized present value of primary deficit $\{Dd_t^1\}$. It is useful because the bond holdings are just conditional expectation of Dd

Define $Dd_t^1 = \sum_{j=0}^{\infty} \delta^{1,j} \frac{u'(C_{t+j}^1)}{u'(C_t^1)} (C_{t+j}^1 - D_{t+j}^1)$ as realized present value of primary deficit.

Then we have

$$Dd_t^1 = \delta^{1,t} \frac{u'(C_{t+1}^1)}{u'(C_t^1)} Dd_{t+1}^1 + C_t^1 - D_t^1.$$

And we impose Dd at the end of the day is 0 to make these two equations equivalent namely $Dd_T^1 = 0$.

And we can solve for Dd backwards from $Dd_T^1 = 0$ given that we have got sequence of consumption and dividends in the above steps.

Step 4 : We solve for $\{B_{t-1}^1(D)\}$ in this step by using the equation:

$$B_{t-1}^1(D_t) = E(Dd_t | D_t = D_t)$$

where bond holds are just function of state D .

To implement it we use

$$B_{t-1}^1(D_t) = \frac{1}{T} \sum_{t=1}^T Dd_t^1 I(D_t)$$

where $I(D_t)$ is the indicator function. This could also be regarded as run the regression of Dd to indicator functions, which is the core idea of PEA. Notice that conditional expectation is actually the average over states. However due to the fact that we have a iid world which is definitely ergodic, we just use the average over time to estimate the conditional expectation by the property of ergodicity.

Technically speaking we are not using PEA because we are not iterating on parameters, which is not necessary in our case. We are not relying on the approximant of the right hand side of euler equations as the typical steps do in PEA thanks to the complete market thing gives us the formula to solve for debt and Bd .

A.2 Algorithm for two agents two shares with diverse belief in complete market

In this case every steps are same except that λ is not constant any more. We will have a sequence $\{\lambda_t\}$ because of the diverse probability, which follows

$$\alpha_{t-1}(D_t) \lambda_{t-1} = \lambda_t$$

where $\alpha_{t-1}(D_t) = \frac{prob_{t-1}^2(D_t)}{prob_{t-1}^1(D_t)}$

Another difference lies in step 4 because in this case bond holding is not only the function of D but also a function of λ_{t-1} . For example

$$B_{t-1}^1(D_t) = E(Dd_t | D_t = D_t, \lambda_{t-1})$$

So we need to run the regression of Dd on both D and λ_{t-1} . Actually for the case in A.1 you can also think of λ as a state. But it is neglected in the regression because it is a just constant and has been taken care of by the constant in the regression.

Explicitly the best way to write Dd as the function of the two states are as follows:

$$Dd_t^l = (\alpha_0^h + \alpha_1^h \lambda_{t-1}) I_{D^h}(D_t) + (\alpha_0^l + \alpha_1^l \lambda_{t-1}) I_{D^l}(D_t).$$

Clearly this regression could be run separately both for high and low.

A.3 Algorithm for two agents two shares in incomplete market

We first lay out the F.O.Cs for this case.

$$\text{Either } \frac{1}{c_t^1} P_t^A = \delta^1 E_t \left(\frac{1}{c_{t+1}^1} (P_{t+1}^A + \frac{1}{2} D_t (1 - \tau^{1,A})) \right) \text{ and } S_t^{1,A} > 0$$

$$\text{or } \frac{1}{c_t^1} P_t^A > \delta^1 E_t \left(\frac{1}{c_{t+1}^1} (P_{t+1}^A + \frac{1}{2} D_t (1 - \tau^{1,A})) \right) \text{ and } S_t^{1,A} = 0$$

$$\text{Either } \frac{1}{c_t^1} P_t^H = \delta^1 E_t \left(\frac{1}{c_{t+1}^1} (P_{t+1}^H + \frac{1}{2} D_t (1 - \tau^{1,H})) \right) \text{ and } S_t^{1,H} > 0$$

$$\text{or } \frac{1}{c_t^1} P_t^H > \delta^1 E_t \left(\frac{1}{c_{t+1}^1} (P_{t+1}^H + \frac{1}{2} D_t (1 - \tau^{1,H})) \right) \text{ and } S_t^{1,H} = 0$$

$$\text{Either } \frac{1}{c_t^2} P_t^A = \delta^2 E_t \left(\frac{1}{c_{t+1}^2} (P_{t+1}^A + \frac{1}{2} D_t (1 - \tau^{2,A})) \right) \text{ and } S_t^{2,A} > 0$$

$$\text{or } \frac{1}{c_t^2} P_t^A > \delta^2 E_t \left(\frac{1}{c_{t+1}^2} (P_{t+1}^A + \frac{1}{2} D_t (1 - \tau^{2,A})) \right) \text{ and } S_t^{2,A} = 0$$

$$\text{Either } \frac{1}{c_t^2} P_t^H = \delta^2 E_t \left(\frac{1}{c_{t+1}^2} (P_{t+1}^H + \frac{1}{2} D_t (1 - \tau^{2,H})) \right) \text{ and } S_t^{2,H} > 0$$

$$\text{or } \frac{1}{c_t^2} P_t^H > \delta^2 E_t \left(\frac{1}{c_{t+1}^2} (P_{t+1}^H + \frac{1}{2} D_t (1 - \tau^{2,H})) \right) \text{ and } S_t^{2,H} = 0$$

In equilibrium, there are two types of cases in term of constraint binding. We now describe these two cases in detail.

Case 1: Unconstrained Asset Holding. If the choices of both agents are unconstrained.

The system of equations is overdetermined for consumption. We need to rewrite the equivalent equations to overcome this problem. Toward this end, we rewrite agent 1's F.O.Cs as the following

$$\text{Either } P_t^A g(S_t^{1,A}) = \delta^1 E_t \left(\frac{c_t^1}{c_{t+1}^1} (P_{t+1}^A + \frac{1}{2} D_t (1 - \tau^{1,A})) g(S_t^{1,A}) \right) \text{ and } S_t^{1,A} > 0$$

or $P_t^A g(S_t^{1,A}) > \delta^1 E_t(\frac{c_t^1}{c_{t+1}^1}(P_{t+1}^A + \frac{1}{2}D_t(1 - \tau^{1,A}))g(S_t^{1,A}))$ and $S_t^{1,A} = 0$

Either $P_t^A = \delta^2 E_t(\frac{c_t^1}{c_{t+1}^1}(P_{t+1}^A + \frac{1}{2}D_t(1 - \tau^{1,A})))$ and $S_t^{1,A} > 0$

or $P_t^A > \delta^1 E_t(\frac{c_t^1}{c_{t+1}^1}(P_{t+1}^A + \frac{1}{2}D_t(1 - \tau^{1,A})))$ and $S_t^{1,A} = 0$

We call these F.O.Cs as ajusted F.O.Cs in the sense that they are equivalent variations of the original ones.

We approximate the expectation with exponentiated log linear parameterized expectation. The state variables are stock holdings at the beginning of the period and dividend at the current period.

$$\begin{aligned}
& E_t(\frac{c_t^1}{c_{t+1}^1}(P_{t+1}^A + \frac{1}{2}D_t(1 - \tau^{1,A}))g(S_t^{1,A})) \approx \Phi_1^A(S_{t-1}, D_t; \beta_1^A) \\
& = \exp(\beta_1^A(1)constant_1^A + \beta_2^A(2)\log(S_{t-1}^{1,A}) + \beta_2^A(3)\log(S_{t-1}^{1,H}) + \beta_2^A(4)\log(D_t)) \\
& \quad E_t(\frac{1}{c_{t+1}^H}(P_{t+1}^H + \frac{1}{2}D_t(1 - \tau^{1,H}))) \approx \Phi_1^H(S_{t-1}, D_t; \beta_1^H) \\
& = \exp(\beta_1^H(1)constant_1^H + \beta_1^H(2)\log(S_{t-1}^{1,A}) + \beta_1^H(3)\log(S_{t-1}^{1,H}) + \beta_1^H(4)\log(D_t)) \\
& \quad E_t(\frac{c_t^2}{c_{t+1}^2}(P_{t+1}^A + \frac{1}{2}D_t(1 - \tau^{2,A}))) \approx \Phi_2^A(S_{t-1}, D_t; \beta_2^A) \\
& = \exp(\beta_2^A(1)constant_2^A + \beta_2^A(2)\log(S_{t-1}^{1,A}) + \beta_2^A(3)\log(S_{t-1}^{1,H}) + \beta_2^A(4)\log(D_t)) \\
& \quad E_t(\frac{1}{c_{t+1}^H}(P_{t+1}^H + \frac{1}{2}D_t(1 - \tau^{2,H}))) \approx \Phi_2^H(S_{t-1}, D_t; \beta_2^H) \\
& = \exp(\beta_2^H(1)constant_2^H + \beta_2^H(2)\log(S_{t-1}^{1,A}) + \beta_2^H(3)\log(S_{t-1}^{1,H}) + \beta_2^H(4)\log(D_t))
\end{aligned}$$

Then consumption is obtained from

$$\frac{c_t^2}{c_t^1} = \frac{\Phi_1^H}{\Phi_2^H}$$

and

$$D_t = \mu^1 c_t^1 + \mu^2 c_t^2$$

We then get P_t^H from agent 2's F.O.C with respect to H share or agent 1's F.O.C with respect to H share once we have consumption. We also get P_t^A from agent 2's ajusted F.O.C. After we have P_t^A , we apply the F.O.C of agent 1 for

A share to get $S_t^{1,A}$. Then $S_t^{2,A}$ is got from A share market clear condition. Then we obtain $S_t^{1,H}$ by using agent 1's budget constraint. In the end, we get $S_t^{1,H}$ by applying H share market clear condition.

Case 2: Constrained Asset Holding. If the choices of either agent is constrained, then the some variations are required. We lay out these variations subcase by subcase.

1) $S_t^{1,A} < 0$ and $S_t^{1,H} < 0$

Then we impose $S_t^{1,A} = 0$, $S_t^{2,A} = 2$, $S_t^{1,H} = 0$ and $S_t^{2,H} = 2$, since we assume that population of agent 1 and agent 2 are equal to the half of total population. Then we employ agent 2's F.O.C with respect to H share and his budget constraint to obtain c_t^2 and P_t^H . P_t^A is got from agent 2's adjusted F.O.C with respect to A share. Finally, c_t^1 again is obtained through goods market clear.

2) $S_t^{1,A} < 0$ and $S_t^{1,H} > 2$

We impose $S_t^{1,A} = 0$, $S_t^{2,A} = 2$, $S_t^{1,H} = 2$ and $S_t^{2,H} = 0$. The procedure is same as one in the just previous case except that we are employing agent 1's F.O.C with respect to H share rather than agent 2's.

3) $S_t^{1,A} < 0$ and $0 \leq S_t^{1,H} \leq 2$

Then we impose $S_t^{1,A} = 0$ and $S_t^{2,A} = 2$. Other steps in the unconstrained case remain.

4) $S_t^{1,A} > 2$ and $S_t^{1,H} < 0$

We impose $S_t^{1,A} = 2$, $S_t^{2,A} = 0$, $S_t^{1,H} = 0$ and $S_t^{2,H} = 2$. We then get P_t^A from agent 1's adjusted F.O.Cs with respect to A share. The rest steps are the same with subcase 1.

5) $S_t^{1,A} > 2$ and $S_t^{1,H} > 2$

We impose $S_t^{1,A} = 2, S_t^{2,A} = 0, S_t^{1,H} = 2$ and $S_t^{2,H} = 0$. We get P_t^A from agent 1's adjusted F.O.Cs with respect to A share. The rest steps are the same with subcase 2.

6) $S_t^{1,A} > 2$ and $0 \leq S_t^{1,H} \leq 2$

We impose $S_t^{1,A} = 2$ and $S_t^{2,A} = 0$. We get P_t^A from agent 1's adjusted F.O.Cs with respect to A share. Other steps in the unconstrained case remain.

7) $0 \leq S_t^{1,A} \leq 2$ and $S_t^{1,H} < 0$

We impose $S_t^{1,H} = 0$ and $S_t^{2,H} = 2$. We get P_t^A and $S_t^{1,A}$ from two agents' adjusted F.O.Cs with respect to A share. The rest steps are the same with subcase 1.

8) $0 \leq S_t^{1,A} \leq 2$ and $S_t^{1,H} > 2$

We impose $S_t^{1,H} = 2$ and $S_t^{2,H} = 0$. We get P_t^A and $S_t^{1,A}$ from two agents' adjusted F.O.Cs with respect to A share. The rest steps are the same with subcase 2.

After we have listed the calculation procedure for equilibrium. We now give a full description of the algorithm step by step.

Step 1: Draw N series of T periods of shocks using a random number generator from log normal distribution $\log \epsilon_t^d \sim i.i.N(-\frac{s_d^2}{2}, s_d^2)$, where s_d is calibrated by Chinese stock market data. Provide an initial guess of β_i^j for these parameterized expectations $\Phi_i^j(S_{t-1}, D_t; \beta_i^j) \forall i \in 1, 2 \forall j \in A, H$.

Step 2: Solve the system of equations to obtain the realized endogenous variables such as consumption, share holdings of each agent and prices through the methods described above in the constrained case and unconstrained case. These variables are $N \times T$ matrices.

Step 3: Compute for the realized integrand in the expectation by plugging the realized consumption, share holding, dividend tax and prices in the integrand

Step 4: Reshape the $N \times T$ realized integrand matrix and parameterized expectation matrix to $N * T$ vectors. Run a nonlinear least square regression of realized integrands on parameterized expectations and then get a new guess for the parameters β^{new} . Check if β^{new} and β^{old} are the same or close enough. Otherwise, do step 5.

Step 5: Update the parameter by $\beta(t+1) = \mu\beta(t) + (1-\mu)\beta^{new}$, where μ is between 0 and 1. To force the algorithm to take small steps, we set $\mu = 0.2$.

Step 6: Iterate on β until it converges.

To get a reasonable initial guess basically we employ the idea of homotopy suggested in Lorenzoni and Marcat (1998) in a way that we first solve for one agent one asset model, get the solution for parameters of expectation as the initial guess for parameterized expectation in two agents one asset model, and then again calculate the solution as initial value in two agents two asset model. We do it in this way because it's well known that PEA method is not global convergent and it requires quite good initial parameter values.

To do the PEA with diverse beliefs, we need to implement an additional measure change step which ensures that we run a regression of subjective realized integrand on subjective parameterized expectation. The realized integrands for example $\frac{1}{c_{t+1}}(P_{t+1}^A + \frac{1}{2}D_t(1 - \tau^{1,A}))$ multiplied by $\frac{f^{sub}(D_t)}{f^{ob}(D_t)}$, where $f^{sub}(D_t)$ is subjective distribution of D_t at time t and $f^{ob}(D_t)$ is objective, are subjective realized integrands. And this ratio of density function or measure change is

$$\frac{f^{sub}(D_t)}{f^{ob}(D_t)} = \frac{\frac{1}{a_i D_{t-1}} f(\epsilon_t^d)}{\frac{1}{a_i D_{t-1}} f(\epsilon_t^d)} = \frac{a}{a_i}$$

A.4 A Learning Model with Diverse Beliefs and Dividend Taxes

This section extends the benchmark learning model with diverse beliefs and dividend taxes. The dividend and consumption growths still follow the same processes as equation (20) and (21). There are two types of agents, one is relative optimistic about fundamental growth and the other is relative pessimistic.

Agent i 's maximization problem for $i = o$ or p is

$$\begin{aligned} & \max E_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} & (24) \\ \text{s.t. } & C_t^i + P_t^A S_t^{A,i} + P_t^H S_t^{H,i} = (P_t^A + (1 - \tau^{i,A})D_t)S_{t-1}^{A,i} + (P_t^H + (1 - \tau^{i,H})D_t)S_{t-1}^{H,i} + Y_t \\ & 0 \leq S_t^{A,i} \text{ \& } 0 \leq S_t^{H,i} & (25) \end{aligned}$$

The subjective belief \mathcal{P}^i is the same as \mathcal{P} in section except that agent i believes fundamental growth at rate of a^i instead of a . The first order conditions are

$$(C_t^i)^{-\gamma} P_t^A \geq \delta E_t^{\mathcal{P}^i} ((C_{t+1}^i)^{-\gamma} (P_{t+1}^A + (1 - \tau^{i,A})D_{t+1})) \text{ with equality if } S_t^{A,i} > 0$$

$$(C_t^i)^{-\gamma} P_t^H \geq \delta E_t^{\mathcal{P}^i} ((C_{t+1}^i)^{-\gamma} (P_{t+1}^H + (1 - \tau^{i,H})D_{t+1})) \text{ with equality if } S_t^{H,i} > 0$$

We then can define the subjective expectations of risk-adjusted stock price growth as

$$\beta_t^{i,A} \equiv E_t^{\mathcal{P}^i} \left[\left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} (P_{t+1}^A) \right] \quad (26)$$

$$\beta_t^{i,H} \equiv E_t^{\mathcal{P}^i} \left[\left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} (P_{t+1}^H) \right] \quad (27)$$

And agent i updates $\beta_t^{i,A}$ and $\beta_t^{i,H}$ according to the same adaptive learning schemes as (22) and (23). The consumption good market clearing condition is

$$C_t = C_t^o + C_t^p = 2Y_t + 2D_t$$

Assumption 1 allows us to have the following approximations

$$E_t^{\mathcal{P}^i} \left[\left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} \right] = E_t^{\mathcal{P}} \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \text{ for } i = o, p$$

The pricing equations according to Adam and Marcet (2011) are

$$P_t^A = \max_{i \in o, p} \frac{\delta (a^i)^{1-\gamma} \rho_\epsilon}{1 - \delta \beta_t^{i,A}} (1 - \tau^{i,A}) D_t,$$

$$P_t^H = \max_{i \in o, p} \frac{\delta (a^i)^{1-\gamma} \rho_\epsilon}{1 - \delta \beta_t^{i,A}} (1 - \tau^{i,A}) D_t,$$

Parameters	Value
a^o	1.0024
a^p	1.0004
$\tau^{o,A}$	0.05
$\tau^{p,A}$	0.10
$\tau^{o,H}$	0.20
$\tau^{p,H}$	0.10

Table 5: Parameters Values for Learning Model

The new parameter values are given in table .The simulated stock prices of A share and H share is in figure, and the simulate AH premium in figure.The similar share of AH premium compared with it in figure confirms that different beliefs about capital gains are dominate factor in generating AH premium relative to diverse beliefs and dividend taxes.

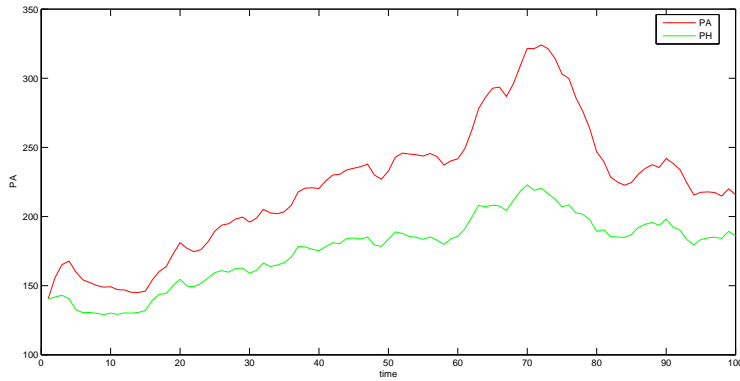


Figure 7: Simulated Stock Prices of A Share and H Share

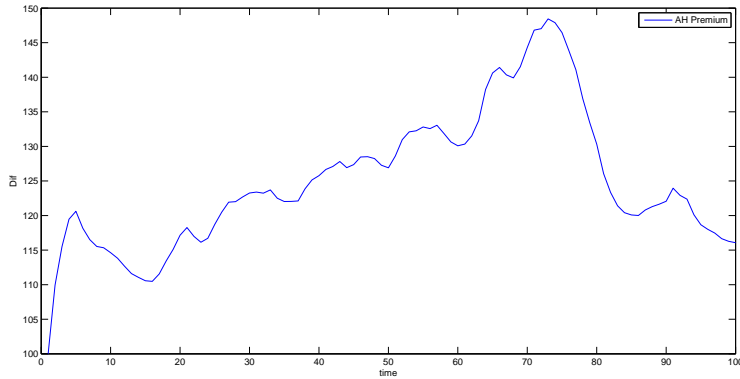


Figure 8: Simulated AH Premium

A.5 Differentiable Projection Facility

The function ω used in the differentiable projection facility is

$$\omega(x) = \left\{ \begin{array}{ll} \beta & \text{if } x \leq \beta^L \\ \beta^L + \frac{\beta - \beta^L}{\beta + \beta^U - 2\beta^L}(\beta^U - \beta^L) & \text{if } \beta^L < x \leq \beta^U \end{array} \right\}$$

Clearly ω is continuous; the only point where continuity is questionable is at $x = \beta^L$, but it is easy to check that

$$\lim_{x \nearrow \beta^L} \omega(x) = \lim_{x \searrow \beta^L} \omega(x) = \beta^L$$

$$\lim_{x \nearrow \beta^L} \omega'(x) = \lim_{x \searrow \beta^L} \omega'(x) = 1$$

$$\lim_{x \rightarrow \infty} \omega(x) = \beta^U$$

In our numerical applications, we choose β^U so that the implied PD ratio never exceeds $U^{PD} = 600$ and $\beta^L = \delta^{-1} - 2(\delta^{-1} - \beta^U)$.

A.6 Data Sources

Our data set for Chinese stock market price, dividend, Heng Seng China AH premium index, Heng Seng China A index and Heng Seng China H index are downloaded from Wind Financial Database (<http://www.wind.com.cn>). The daily price series has been transformed into a weekly series by taking the index value of the last day of the considered week.

Our data set for Chinese consumption and CPI are downloaded from Chung, Chen, Waggoner and Zha (2015) (<http://www.tzha.net/code>).