Wealth Accumulation, On the Job Search and Inequality

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Abstract

To study the equilibrium interactions between wealth accumulation and labor market search, this paper constructs a model where individuals can accumulate non-contingent assets under a borrowing limit, all workers can search for jobs, and search is directed. On-the-job search generates a wage ladder, which affects inequalities in earnings, wealth and consumption. Employed workers have incentive to save as a precaution for exogenous separation into unemployment. In the reverse direction, wealth and earnings affect search decisions by changing the optimal tradeoff between the wage and the matching probability. The calibrated model reveals that wealth significantly reduces a worker's transition rate from unemployment to employment and from one job to another. Moreover, search frictions increase wealth inequality significantly by increasing the mass of wealthy individuals and lengthening the right tail of the wealth distribution. However, the effect of wealth on job search widens frictional wage dispersion by only a small amount. In addition, on-the-job search is important for frictional wage dispersion.

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1. Introduction

For a typical individual, labor income is the chief source of income. Even when the aggregate economy is stable, a worker's labor income fluctuates because of changes in jobs and the employment status, etc. A worker is unable to perfectly insure against such risks when the market is incomplete for assets contingent on workers' labor market outcomes. However, a worker can accumulate non-contingent assets to partially insure against earnings risks. In addition, a worker can modify job search strategies in order to reduce income risks, e.g., by searching for jobs that are relatively easy to obtain. How do job search and wealth accumulation interact in the equilibrium? What are the implications of these interactions on inequalities in income, wealth and consumption? The objective of this paper is to address such questions with a model where individuals can accumulate assets under a borrowing limit and search in the labor market both off and on the job.

It is well understood that labor income risks affect individuals' consumption and wealth because of incomplete asset markets. Recent literature also documents the effect in the reverse direction; that is, an individual's wealth affects labor market outcomes. For example, Chetty (2008) reports that individuals who receive large severance payments stay unemployed for longer periods of time. Herkenhoff et al. (2015) show that increasing individuals' ability to borrow in the U.S. increases unemployment duration significantly. This positive effect of wealth on unemployment duration also exists in France (Algan et al., 2003), Germany (Bloemen and Stancenelli, 2001), and Denmark (Lentz and Tranbaes, 2005). Moreover, the wealth effect is not limited to the transition from unemployment to employment. Lise (2013) shows that wealthier employed workers have lower transition rates from one job to their next job. Thus, an individual's earnings process has an endogenous component that is affected by wealth. The two-way interactions between job search and consumption/savings decisions deserve special attention in research.

A model that focuses on these interactions is also useful for understanding how labor market frictions and job search affect inequality. In particular, Budria et al. (2002) report that wealth is very unequally distributed across U.S. households. This distribution has positive skewness, a short and fat left tail, and a long right tail. How much of such wealth inequality can be accounted for by labor market frictions is a question that has not yet been addressed in detail. On wage inequality, an important issue is how much wage inequality search frictions alone can generate. Hornstein et al. (2011) have demonstrated that such "frictional" wage inequality is small in a variety of search models: the ratio of the average to the minimum wage is about 1.04 in the equilibrium. By interacting with job search decisions, wealth accumulation can in principle affect frictional wage inequality.

In the model in this paper, individuals are risk averse. They can save or borrow in terms of non-contingent assets but face a borrowing limit. To focus on the implications of search frictions on inequality, we assume that all workers have the same productivity and all jobs are the same. Thus, all wage dispersion in the model is frictional in the sense that it is generated by search frictions. Both employed and unemployed workers can search. The possibility of failure to match generates labor income risks. Over time, workers differ in the history of search outcomes. This endogenous heterogeneity induces dispersion in earnings, wealth and consumption. Moreover, this endogenous heterogeneity affects workers' search decisions. Workers who differ in the current wage and wealth face different trade-offs between consumption smoothing and future wage gains. They optimally choose to search for different wages. Firms incur a cost to create vacancies competitively in submarkets to *direct* workers' search. Each submarket offers a distinct combination of wage and the matching probability to tailor to particular applicants. In the equilibrium, jobs with a higher wage have a lower matching probability for an applicant.

In this context, wealth accumulation and labor market search interact as follows. For individuals with high wealth, labor income risk is not very significant because they are able to smooth consumption by decumulating assets. Hence, they prefer to apply for a high wage and face a lower matching probability. In contrast, for individuals with asset levels at or closer to the borrowing limit, consumption smoothing is a very important issue. They find it optimal to apply for a low wage that has a high matching probability. In turn, the dependence of search decisions on wealth induces different paths of earnings which affect the dynamics of wealth. This mechanism can qualitatively produce the aforementioned features that workers with higher wealth or borrowing ability spend more time making the transition from unemployment to employment and from one job to another.

To assess the quantitative importance of the mechanism, we calibrate the baseline model and compare the results with two benchmarks in the literature. One benchmark is a "no-search" model similar to Aiyagari (1994), where the labor market is Walrasian but individuals face employment risks and a borrowing limit. The probability of becoming employed and the probability of losing employment are set to be equal to the average counterparts in the baseline model. The other benchmark is a "no-wealth" model where individuals are hand-to-mouth. This benchmark differs from the canonical search model (e.g., Diamond, 1981, Mortensen, 1982, and Pissarides, 2000) in that search is directed, workers can search on the job and workers are risk averse.

The main quantitative findings are as follow. Search frictions induce significant inequality in wealth. Relative to the no-search model, the baseline model generates a significantly longer right tail and larger mass of wealthy individuals. Search frictions increase wealth inequality for two reasons. First, even high-wage workers continue to search for wage increases in order to build up wealth to insure against exogenous separation into unemployment. This force extends the upper tail of the wealth distribution. Notice that the same motive of self-insurance exists in the no-search model because exogenous job separation exists there too. However, the mechanism is muted in the no-search equilibrium because there is no wage dispersion for workers to search for. Second, workers with low wealth are willing to lower their target wage for search in order to become employed quickly. If these workers fail to move up the wage ladder after getting their first job, their low wage makes their wealth fall quickly. This force stretches the lower tail of the wealth distribution.

In the reverse direction, wealth and current earnings affect job search decisions and frictional wage dispersion. Using the model generated data, we regress the transition rates from unemployment to employment and from one job to another on wealth and current earnings. The regression coefficients are negative and statistically significant. Thus, a worker with higher wealth or earnings is less likely to move from unemployment to employment or from one job to another. Moreover, the effect of wealth increases frictional wage inequality, but this increase is small. The mean-min wage ratio is 1.085 in the baseline model and 1.062 in the no-wealth benchmark. The presence of wealth accumulation increases frictional wage dispersion primarily by inducing workers with low wealth to lower their search target, as described above. This effect is small because of the calibration. In both the baseline and the two benchmark models, the calibration requires the average job-finding probability of unemployed workers to match the one in the U.S. data. Since this job-finding probability is high, those individuals for whom wealth is an important determinant of job search decisions have a relatively small mass in the equilibrium.

On-the-job search is important for frictional wage dispersion. Although the mean-min wage ratio in the baseline, 1.085, is substantially lower than the empirically observed ratio, it is higher than the ratio, 1.04, that Hornstein et al. (2011) have calculated for most search models where on-the-job search and wealth accumulation are not allowed. Allowing for wealth accumulation but excluding on-the-job search, Krusell et al. (2010) find that the mean-min wage ratio is even smaller, 1.023. On-the-job search widens wage dispersion by endogenously creating a wage ladder. However, this wage ladder is short because search is directed and, as mentioned above, the lowest wage in the equilibrium is relatively high in order to match the unemployed workers' job-finding probability.[1]

Savings provide significant self insurance against the earnings risk. In the no-search model, the Gini coefficient in consumption is 51.8% of that in earnings. That is, 48.2% of the earnings risk is self insured by accumulating wealth. Directed search provides additional insurance by reducing the Gini coefficient in consumption to 46.6% of that in earnings. Finally, we investigate the effects of the borrowing limit and the interest rate on inequality (see section 4.4 for a summary and Appendix A for the analysis).

Our work builds on several strands of the literature. Earlier models by Andolfatto (1996) and Merz (1995) embed undirected search into intertemporal macro models with

[1] If search is undirected, as in Burdett and Mortensen (1998), on-the-job search can generate wider wage dispersion by reducing the lowest equilibrium wage (see Hornstein et al., 2011). However, such dispersion is not robust to directed search, because firms offering such low wages cannot attract applicants.

capital accumulation to study business cycles. Allowing for both directed and undirected search, Shi and Wen (1999) evaluate taxes and subsidies with capital accumulation. In these models, idiosyncratic risks generated by labor search are completely smoothed either within large households or in a perfect insurance market. Moreover, individuals do not face a borrowing limit. Lentz and Tranaes (2005) and Lentz (2009) analyze the optimal unemployment insurance when wealth affects unemployed workers' search effort. Since these models do not allow for on-the-job search, the equilibrium has a single wage rate. There are also papers that analyze how hidden savings affect unemployment insurance when there is moral hazard, e.g. Werning (2001) and Kocherlakota (2004).

A paper more closely related to ours is Krusell et al. (2010). These authors integrate labor search into a model of precautionary savings with income shocks in the style of Bewley (1977), Huggett (1993) and Aiyagari (1994). Our model differs from Krusell et al. (2010) in two main aspects. First, we allow for on-the-job search. By generating a wage ladder, on-the-job search incorporates additional volatility in the income process, thus enriching the interactions between job search and wealth accumulation. Second, search is directed in our model but undirected in Krusell et al. (2010). Directed search is intuitive and realistic.[2] In particular, it enables workers to adjust their wage target for search according to their wealth. In addition, directed search simplifies the analysis and computation by making the equilibrium block recursive (see Shi, 2009). Namely, individuals' decisions and the market tightness do not depend on the large dimensional object – the distribution of workers across individual states. With the much reduced state space, we can compute the dynamic equilibrium exactly, rather than approximate it as in Krusell et al. (2010).

Another closely related paper is Lise (2013), who also studies how on-the-job search affects wealth accumulation. A major difference from our model is that Lise (2013) assumes an exogenous distribution of wages from which workers draw offers. The offer distribution is endogenous in our model and this endogeneity is critical for our objective. Rather than matching wage dispersion in the model with the data, we try to understand how much

[2]For example, in a survey data in the U.S., Hall and Krueger (2008) document that 84 percent of white, male, non-college workers either "know exactly" or "had a pretty good idea" about how much their current job would pay before the job interviews.

wealth inequality can be generated by search frictions endogenously. Another substantial difference is that search is undirected in Lise (2013) but directed in our model. The importance of this difference is discussed above in the comparison of our model with Krusell et al. (2010). As an illustration of how this difference is relevant, consider two unemployed workers: one has high wealth and just lost a high-wage job, while the other has low wealth and has been unemployed for a long time. The two workers have the same reservation wage in Lise (2013) and, hence, accept the same set of offers. In our model, in contrast, the poor unemployed worker will choose to search for a lower wage to become employed faster than the rich counterpart. This implication of our model seems to accord well with the empirical evidence discussed earlier on how wealth affects unemployment duration.

Finally, this paper is closely related to Herkenhoff (2015) in the spirit but differs in the objectives. He uses a directed search model to study how increasing the access to credit card debt impacts unemployment duration and job recoveries after recessions. The main parts of his analysis assume that only unemployed workers can search, although he discusses in an appendix how to extend the analysis to allow for on-the-job search. In contrast, we are particularly interested in the case with on-the-job search.

The paper is organized as follows. In section 2, we introduce the model environment, analyze individuals' optimal decisions and characterize market tightness. In section 3, we define the equilibrium and explain why it is block recursive. Section 4 calibrates the model to study the quantitative implications. Section 5 concludes. The appendices provide additional materials, including counter-factual exercises.

2. Model of Consumption, Savings and Search

2.1. Environment of the model

Time is discrete and lasts forever. There is a measure of risk averse workers with utility in each period being given by $U : \mathbb{R}_+ \to \mathbb{R}$, where $U'(c) \in (0, \infty)$ and $U''(c) \in (-\infty, 0)$ $\forall c \in \mathbb{R}_+$, and $U'(0) = \infty$. A worker's discount factor is $\beta \in (0, 1)$. Workers can accumulate non-contingent assets, denoted $a \in A = [\underline{a}, \bar{a}]$. The lower bound on asset holdings, $\underline{a}$, can be negative, in which case it is a borrowing limit. The upper bound $\bar{a}$ can be chosen properly so that it is not binding. The net rate of return on assets is r, which is determined in the world market and taken as given in this model. We assume $\beta(1+r) < 1$ in order to ensure existence of an equilibrium in the presence of precautionary savings.[3]

In each period, a worker is endowed with one indivisible unit of labor, and so a worker is either employed full-time or unemployed. An employed workers obtains wage $w \in W \equiv [\underline{w}, \bar{w}]$, and an unemployed worker has home production $b \geq 0$. Thus, ex ante identical workers become endogenously heterogeneous ex post in three aspects: the employment status (employed or unemployed), current earnings including home production, and the wealth level. In each period, an unemployed worker is able to search for a job with probability λ_u. An employed worker is able to search on the job with probability λ_e, which we will use for calibration and counter-factual exercises.

The measure of firms is determined by competitive entry. Firms are risk neutral and have the same discount factor β as workers. The production technology has constant returns to scale in jobs, and so a firm treats the jobs independently. To simplify the terminology, we refer to a job as a firm. All jobs have the same quality. Each filled job yields a constant stream of output, $y > b$, and the production cost is normalized to 0. The vacancy cost per period is $k > 0$. Firms commit to offers, but workers can quit a job at anytime. In addition, firms cannot respond to the employee's outside offers. If an employed worker receives a better offer, the worker quits the current job.[4] In addition to such endogenous separation, an old match can be destroyed exogenously in a period with probability $\delta \in (0, 1)$, in which case the worker becomes unemployed. Separation shocks are *iid* across old matches and time. To reduce repetitive accounting, we assume that a newly formed match is not subject to exogenous separation in the same period.

The labor market is organized into a continuum of submarkets indexed by the wage offer w and the applicants' wealth a. Firms choose the submarket to enter to create vacancies.

[3] As shown by Aiyagari (1994), when individuals face income risks and a borrowing limit, the precautionry saving motive would induce them to accumulate infinite wealth if $\beta(1+r) < 1$ were violated.

[4] See Postel-Vinay and Robin (2002) for an undirected search model where firms can match an employee's outside offers.

Workers observe the offers in all submarkets before choosing the submarket to search. In this sense, search is directed. Each submarket (w, a) implies a tightness $\theta(w, a)$, which is the ratio of vacancies to applicants in the submarket. In principle, the tightness might also depend on the aggregate state of the economy. For the reason that will become clear in section 3, this dependence does not arise in the equilibrium.

Matching is random inside each submarket. The matching technology has constant returns to scale in the measure of applicants and vacancies. In any submarket with tightness θ, the matching probability is $p(\theta)$ for an applicant and $q(\theta)$ for a vacancy, where $p(\theta) = \theta q(\theta)$ because of constant returns to scale. Also, the matching probabilities satisfy the standard assumptions: $p(\theta), q(\theta) \in [0, 1]$, $p'(\theta) \in (0, \infty)$, $q'(\theta) \in (-\infty, 0)$ and $p''(\theta) < 0$ for all θ. Although $p(\theta)$ and $q(\theta)$ are exogenous functions of θ, the tightness function $\theta(w, a)$ is endogenously determined by firms' entry and workers' search decisions. Thus, the matching probabilities are endogenous functions of (w, a).

Since a submarket is indexed by (w, a), the offer w is given to a worker matched in the submarket conditional on the worker's wealth being a. Such conditioning simplifies the description of the equilibrium, but the analysis does not require firms to be able to directly observe an applicant's wealth level. Rather, as discussed in section 3.2, a recruiting firm has incentive to elicit information about an applicant's wealth and an applicant has incentive to truthfully report the wealth level. We will also discuss how firms can use an applicant's resume to precisely infer about the applicant's wealth.

Each period is divided into four stages: (i) production, (ii) consumption and savings, (iii) search and matching, and (iv) separation. In the production stage, all existing matches produce, and earnings (including home production) are realized. In the next stage, individuals choose savings and consumption. In the third stage, the opportunity to search is realized according to probability λ_u for each unemployed worker and λ_e for each employed worker. A worker who receives the search opportunity chooses the submarket (w, a) to search. Matches are formed according to the probabilities specified above. A worker who fails to find a new match will start the following period at their current position with the new wealth level chosen in stage (ii). Finally, in stage (iv), a separation shock destroys a fraction $\delta \in (0,1)$ of old matches.

For the subsections to follow, we define some objects. Let $E \equiv \{e_0, e_1\}$ be the space of the employment status with its Borel sets $\mathcal{E}$, where e_0 denotes unemployment and e_1 employment. The space of asset holdings is $A \equiv [\underline{a}, \bar{a}]$ with its Borel sets $\mathcal{A}$. The space of workers' values at the consumption-saving stage is $V \equiv \left[\underline{V}, \bar{V}\right]$ with its Borel sets $\mathcal{V}$. The space of current earnings is $\bar{W} \equiv [\underline{w}, \bar{w}] \cup b$ with its Borel sets $\mathcal{W}$. A worker's individual state variables are the current employment status $e \in E$, current earnings $w \in \bar{W}$, and asset holdings, $a \in A$. A worker's individual state is $s \equiv (e, w, a) \in S$, where $S \equiv E \times \bar{W} \times A$ with the Borel sets in $\mathcal{S} = \mathcal{E} \times \mathcal{W} \times \mathcal{A}$. Note that current earnings are the current wage if a worker is employed and home production if the worker is unemployed.

The state of the economy is given by $\psi : \mathcal{S} \to [0,1]$, a distribution of workers over employment status, wealth, and asset holdings. Let $\Psi(S, \mathcal{S})$ be the space of distribution functions on the measurable space $(S, \mathcal{S})$. Let $\mathcal{T} : \Psi(S, \mathcal{S}) \to \Psi(S, \mathcal{S})$ be the law of motion for the state of the economy. As explained later, the equilibrium in this model is block recursive in the sense that individuals' optimal decisions, value functions and the market tightness are independent of the aggregate state (distribution). In particular, the market tightness function is $\theta : W \times A \to \mathbb{R}_+$, as introduced earlier.

2.2. Unemployed worker's decisions

Consider an unemployed worker whose wealth at the beginning of a period is a. The worker's value function is $V_u(a)$, where the subscript u indicates the status of unemployment e_0 and home production is suppressed in the notation. The worker's choice problem can be formulated in two stages. In the consumption and savings stage, the worker chooses consumption c and future wealth $\hat{a}$. In the search and matching stage, the worker chooses the target wage $\hat{w}$ to search for, given the wealth level $\hat{a}$. The optimal search decision generates the following *return on search for the unemployed worker*:

$$R_u(\hat{a}) \equiv V_u(\hat{a}) + \max_{\hat{w}} \ \lambda_u p(\theta(\hat{w}, \hat{a})) \left[V(\hat{w}, \hat{a}) - V_u(\hat{a})\right]. \tag{2.1}$$

If the worker is matched at the target wage $\hat{w}$, the worker will start the next period employed, which will yield the value $V(\hat{w}, \hat{a})$ to the worker. If the worker fails to match, the worker will stay unemployed at the beginning of the next period, which will yield the value $V_u(\hat{a})$. The "capital gain" from finding the job $\hat{w}$ is $[V(\hat{w}, \hat{a}) - V_u(\hat{a})]$. The expected capital gain is the maximand in (2.1), where the parameter λ_u is the probability that an unemployed worker is able to search for a job in a period. Denote the optimal search target by the policy function $\tilde{w}_u(\hat{a})$.

The unemployed worker's choices of consumption c and future wealth $\hat{a}$ solve:

$$V_u(a) = \max_{c,\hat{a}} \; [u(c) + \beta R_u(\hat{a})] \tag{2.2}$$

$$\text{s.t.} \quad c + \frac{\hat{a}}{1+r} = b + a \quad \text{and} \quad \hat{a} \geq \underline{a}.$$

The first constraint is the budget constraint and the second constraint the borrowing limit. Denote the optimal choices by the policy functions $c_u(a)$ and $\hat{a}_u(a)$. Rewrite the optimal search choice as a function of the worker's wealth at the beginning of the period instead of at the search stage: $\hat{w}_u(a) \equiv \tilde{w}_u(\hat{a}_u(a))$. Similarly, express the tightness of the submarket searched by the unemployed worker as $\hat{\theta}_u(a) \equiv \theta(\hat{w}_u(a), \hat{a}_u(a))$.

The formulations in (2.1) and (2.2) reveal that a worker's risk aversion and the borrowing limit generate a wealth effect on job search decisions. Consider an unemployed individual with low wealth. Given that the worker is unemployed, he expects that in the future he will find a job with some probability and his income will increase. Since expected income in the future will be higher than current income, the risk averse worker would like to smooth consumption by decumulating wealth to transfer some of the higher future income to the present. However, the borrowing limit constrains the worker's ability to smooth consumption. If the borrowing constraint is binding, the low wealth forces the worker to consume a relatively low amount. If the worker fails to find a job, consumption will likely be even lower in the next period. To reduce the likelihood of such falling consumption, the worker will choose to search for jobs that have relatively high job-finding probabilities. In equilibrium, those jobs are the ones paying lower wages. Thus, workers with lower wealth levels have incentives to apply for lower wages.

2.3. Employed worker's decisions

Consider an employed worker with wage w and wealth a at the beginning of a period. The worker's value function is $V(w,a)$. Again, we formulate the worker's choice problem recursively in two stages. In the search stage, the worker chooses the search target $\hat{w}$ given wealth $\hat{a}$. This choice yields the *return on search for the employed worker*:

$$\begin{aligned} R(w,\hat{a}) \equiv\ & \delta V_u(\hat{a}) + (1-\delta) V(w,\hat{a}) \\ & + \max_{\hat{w}} \lambda_e p(\theta(\hat{w},\hat{a})) \left[V(\hat{w},\hat{a}) - \delta V_u(\hat{a}) - (1-\delta) V(w,\hat{a})\right]. \end{aligned} \tag{2.3}$$

The worker's current job can be destroyed with probability δ, in which case the value for the worker at the beginning of the next period will be $V_u(\hat{a})$. If the current job is not destroyed, staying at the job will yield the value $V(w,\hat{a})$ to the worker. Thus, if the worker does not move to another job, the expected value will be $[\delta V_u(\hat{a}) + (1-\delta) V(w,\hat{a})]$. If the worker moves to a new job at $\hat{w}$, the worker will start the next period with the value $V(\hat{w},\hat{a})$. The worker chooses the target wage $\hat{w}$ to maximize the expected gain from search. Denote the optimal search target by the policy function $\tilde{w}(w,\hat{a})$.

The employed worker's choices of consumption c and future wealth $\hat{a}$ solve:

$$V(w,a) = \max_{c,\hat{a}} \ [u(c) + \beta R(w,\hat{a})] \tag{2.4}$$

$$\text{s.t.} \quad c + \frac{\hat{a}}{1+r} = w + a \quad \text{and} \quad \hat{a} \geq \underline{a}.$$

Denote the optimal choices by the policy functions $c(w,a)$ and $\hat{a}(w,a)$. Rewrite the optimal search target as $\hat{w}(w,a) \equiv \tilde{w}(w,\hat{a}(w,a))$. Similarly, express the tightness of the submarket searched by the employed worker as $\hat{\theta}(w,a) \equiv \theta(\hat{w}(w,a), \hat{a}(w,a))$. As for an unemployed worker, risk aversion and the borrowing limit generate a wealth effect on an employed worker's on-the-job search decision. That is, individuals with low wealth choose to search for lower wages than individuals with high wealth.[5]

[5] $V(w,a)$ is equal to the larger value of the right-hand side of (2.4) and $V_u(a)$. To reduce clutter, we omit this additional maximization problem, without loss of generality. $V(w,a) \geq V_u(a)$ holds for all equilibrium values of (w,a). In particular, starting with any $V(w,a) \geq V_u(a)$, (2.3) implies $R(w,a) \geq V_u(a)$ and $u(c) + \beta R(w,\hat{a}) \geq u(c) + \beta V_u(\hat{a})$ for all $(c,w,\hat{a})$. The comparison between (2.4) with (2.2) verifies $V(w,a) \geq V_u(a)$, indeed.

In addition, given the wealth level, an employed worker's current wage affects the search decision. When an employed worker fails to find a new match, the worker keeps the current job. The higher is the wage at the current job, the smaller is the effect of such match failure on income. Thus, a worker with a relatively high current wage will search for a higher wage. Although this effect is common in models with directed search on the job, it has the new feature here that it interacts with risk aversion and wealth. Because of risk aversion, wage gains from finding a new match yield a diminishing marginal utility of consumption. This weakens the incentive to search for large wage gains if the worker is already wealthy.[6]

2.4. Firms and market tightness

Consider a filled job (w, a) at the beginning of a period, i.e., a filled job paying wage w to a worker whose wealth at the beginning of the period is a. Current profit of the job is $(y - w)$. The worker will be able to search on the job with probability λ_e. The search target will offer wage $\hat{w}\,(w, a)$ with the associated tightness $\hat{\theta}\,(w, a)$. The worker will succeed in getting a new match with probability $p(\hat{\theta}\,(w, a))$, in which case the worker will separate from the current job endogenously. With probability δ, the current job will be destroyed, in which case the worker will separate exogenously. After either separation, the value of the job to the firm will be zero. If neither separation occurs, the continuation value of the job to the firm will be $\beta J\,(w, \hat{a}\,(w, a))$, where $\hat{a}\,(w, a)$ is the worker's future wealth. Thus, the value of the job to the firm satisfies:

$$J\,(w, a) = y - w + \beta\,(1 - \delta)\,[1 - \lambda_e p(\hat{\theta}\,(w, a))]\,J\,(w, \hat{a}\,(w, a))\,. \tag{2.5}$$

Competitive entry of vacancies determines the tightness in each submarket. Consider submarket $(\hat{w}, \hat{a})$. The matching probability for the vacancy is $q\,(\theta\,(\hat{w}, \hat{a}))$. If the firm is matched, production will start in the next period, and the value of the firm is $\beta J\,(\hat{w}, \hat{a})$. The flow cost of posting a vacancy is $k > 0$. For all $(\hat{w}, \hat{a})$, if $\beta J(\hat{w}, \hat{a}) \geq k$, then entry by vacancies will induce such tightness in the submarket that equates the expected value of a vacancy to the cost k. If $\beta J(\hat{w}, \hat{a}) < k$, then no vacancy will enter the submarket. Thus,

[6] Section 4.2 will analyze the features of the policy functions in detail.

competitive entry of vacancies yields

$$q\left(\theta\left(\hat{w},\hat{a}\right)\right)\beta J\left(\hat{w},\hat{a}\right)\leq k \text{ and } \theta\left(\hat{w},\hat{a}\right)\geq 0, \forall\left(\hat{w},\hat{a}\right),$$

where the two inequalities hold with complementary slackness. This condition can be written explicitly as a solution for the tightness:

$$\theta\left(\hat{w},\hat{a}\right)=\begin{cases} q^{-1}\left(\frac{k}{\beta J(\hat{w},\hat{a})}\right) & \text{if } \beta J\left(\hat{w},\hat{a}\right)\geq k \\ 0 & \text{otherwise.}\end{cases} \tag{2.6}$$

Note that $q(\theta)$ is a decreasing function and $p(\theta)$ an increasing function. A submarket with relatively high job-finding probability for an applicant must have a relatively large tightness. Because this submarket has relatively low matching probability for a vacancy, firms enter such a submarket only if a vacancy filled in the submarket has a relatively high value J, as shown by (2.6). To yield such a high value J, the wage offer must be relatively low in this submarket.

3. Equilibrium and the Submarkets

3.1. Equilibrium definition and block recursivity

The aggregate state of the economy is a distribution of workers over the states consisting of the employment status, earnings (home production) and wealth. Given the distribution of workers at the beginning of a period, ψ, individuals' optimal decisions and matching outcomes induce the distribution of workers at the beginning of the next period, $\hat{\psi}$. We omit the characterization of this transition of the aggregate state because it involves cumbersome accounting of the flows of workers between states. An equilibrium can be defined as follows:

Definition 3.1. *An* equilibrium *consists of value functions* (V_u, V, J), *workers' policy functions,* $(\hat{w}_u, c_u, \hat{a}_u)$ *and* $(\hat{w}, c, \hat{a})$, *and the transition function of the aggregate state,* $\mathcal{T}$, *that satisfy the following requirements:*
(i) The value function for unemployed workers, $V_u : A \to \mathbb{R}$, *satisfies (2.2) for all* $a \in A$, *and the corresponding optimal decisions yield the policy functions of search, consumption and future wealth,* $\hat{w}_u : A \to \mathbb{R}$, $c_u : A \to \mathbb{R}_+$ *and* $\hat{a}_u : A \to A$;

(ii) The value function V for employed workers, $V: W \times A \rightarrow \mathbb{R}$, satisfies (2.4) for all $(w, a) \in W \times A$, and the corresponding optimal decisions yield the policy functions of search, consumption and future wealth, $\hat{w}: W \times A \rightarrow \mathbb{R}$, $c: W \times A \rightarrow \mathbb{R}_+$, and $\hat{a}: W \times A \rightarrow A$;
(iii) The firm's value function, $J: W \times A \rightarrow \mathbb{R}$, satisfies (2.5) for all $(\hat{w}, \hat{a}) \in W \times A$;
(iv) The tightness function θ satisfies (2.6) for all $(\hat{w}, \hat{a}) \in W \times A$.
(v) The aggregate state transition, $\mathcal{T}: \Psi(S, \mathcal{S}) \rightarrow \Psi(S, \mathcal{S})$, is consistent with the policy functions and induces the aggregate state in the next period as $\hat{\psi} = \mathcal{T}(\psi)$.

The equilibrium defined above is a *block recursive equilibrium* (BRE), as defined and analyzed by Shi (2009) and Menzio and Shi (2010, 2011). Namely, value functions, policy functions and the market tightness function in the equilibrium are determined by (i)-(iv) independently of the distribution of workers. Block recursivity can be verified as follows. Start with the hypothesis that the value function of a filled job, $J(w, a)$, is independent of the distribution of workers, ψ. Competitive entry of vacancies into submarkets requires the expected value of a filled job to be equal to the vacancy cost. This requirement determines the market tightness function, $\theta(w, a)$, independently of ψ (see (2.6)). Because matching probabilities in a submarket is only a function of the market tightness, they are also independent of ψ. Given these matching probabilities and the contracts, individuals can calculate the present value of a job and make their decisions. These value functions and optimal choices are independent of ψ. In particular, the value function of a filled job to a firm, $J(w, a)$, is independent of ψ, which supports the initial hypothesis.

Directed search and competitive entry of vacancies are critical for block recursivity. Because search is directed, a worker chooses to search in the submarket that features the optimal tradeoff between the gain in value and the matching probability. For this decision, the worker does not need to know the distribution of workers, provided that the matching probabilities are independent of the distribution. Because competitive entry of vacancies drives down expected profit of a vacancy to zero in every viable submarket, the tightness and the matching probabilities are indeed independent of the distribution.

Block recursivity reduces the complexity of the equilibrium substantially. If the equilibrium is not block recursive, then the distribution of workers is a state variable relevant for individuals' decisions. This aggregate state has infinite dimension because earnings and wealth lie in intervals. Moreover, the law of motion of this aggregate state is endogenous because it must be consistent with the flows of workers induced by individuals' optimal decisions. Exactly computing such a non-block recursive equilibrium is not feasible. The approximation in the literature assumes that only a small number of moments of the distribution matter (e.g., Krusell and Smith, 1998, Krusell et al., 2010). In contrast, when the equilibrium is block recursive, individuals' optimal decisions and the market tightness function depend only on the individual state $s = (e, w, a)$ that has a small dimension.

3.2. Indexing submarkets by wage offers and applicants' wealth

For the equilibrium to be block recursive, workers should be able to choose to search in the submarket indexed by not only the wage offer w but also the applicants' wealth a. Let us elaborate on this dependence of the submarkets on a. Why does a firm care about an applicant's wealth as well as the offer? To a recruiting firm, an applicant's wealth affects the value of a filled job by affecting the worker's endogenous separation probability in the future, $p(\hat{\theta}\,(w, a))$, as shown in (2.5). As explained in a worker's decision, risk aversion and the borrowing limit together imply that a worker with low wealth has incentive to search for a job with high matching probability. Because of this effect of wealth on a worker's search decision, a job filled by a worker with low wealth is expected to survive for a short time, given the same wage offer. Since such a filled job yields a low expected present value to a firm, the firm prefers to hire workers with high wealth.

Suppose that firms in each submarket can ask the applicants to show their wealth, they will hire the applicants with the highest wealth. An applicant is not able to show more wealth than what he has. If an applicant chooses to hide some wealth, there is no gain once the worker is hired, because the wage is posted. However, there is risk that the worker will not be hired if other applicants who visit the same firm report their wealth truthfully. Thus, it is incentive compatible for all applicants to show their wealth truthfully. Similarly, two workers who differ in either their current wage or wealth do not choose to enter the same submarket indexed by (w, a).

In light of the above discussion, it may be puzzling why firms in reality do not explicitly elicit information about applicants' wealth. In many states in the U.S., an employer is allowed to gain access to an employee's credit record. Although a credit record may not precisely identify the wealth level, it can be highly correlated with the wealth level. In addition, firms can select applicants based on their resumes, which can be sufficient for revealing applicants' wealth precisely. In the current model, if all workers had the same initial wealth a_0 when they entered the economy, then a resume that perfectly reveals wealth only needs to contain a worker's current employment status, current earnings (or home production) and the duration in which such earnings have lasted. This result can be proven by induction.[7] Although this alternative formulation of the contracting space is more direct, it is omitted here because it increases clutter.

4. Quantitative Analysis

4.1. Calibration

The utility function and the matching probabilities have the following forms:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \quad p(\theta) = \left[1 + (p_0/\theta)^\gamma\right]^{-\frac{1}{\gamma}}, \quad q(\theta) = \frac{p(\theta)}{\theta}.$$

If a submarket has a measure N of applicants and tightness θ, then the measure of vacancies is θN and the measure of matches is:

$$\mathcal{M}(N, \theta N) = p(\theta) N = \frac{N \times (\theta N)}{\left[(p_0 N)^\gamma + (\theta N)^\gamma\right]^{\frac{1}{\gamma}}}.$$

The parameters γ and p_0 are positive and finite. When $\gamma = 1 = p_0$, the matching function is the telephone matching function.

[7] At the time when a worker enters the economy, the worker is unemployed and searches for the wage $\hat{w}_u(a_0)$. At the time of search, firms can calculate the worker's wealth as $\hat{a}_u(a_0)$. At the time of search in the next period, this worker's wealth will be $\hat{a}(\hat{w}_u(a_0), \hat{a}_u(a_0))$ if the worker enters the next period employed, and $\hat{a}_u(\hat{a}_u(a_0))$ if the worker enters the next period unemployed. In both cases, firms can calculate the worker's wealth precisely, because the functions $(\hat{a}, \hat{a}_u, \hat{w}_u)$ are common knowledge. Continuing this precess, firms can infer an applicant's wealth in any arbitrary period.

The model contains twelve parameters whose baseline values and calibration targets are listed in Table 1. The length of a period is one month. We assume that the borrowing limit is $\underline{a} = 0$.[8] The discount factor $\beta = 0.996$ is used to match an annual discount rate of approximately 5%. The curvature of the utility function is set $\sigma = 2$, a standard value in macro models. As in Shi (2016), the average elasticity of the job-finding probability of unemployed workers with respect to average market tightness for unemployed workers lies between 0.27 and 0.50. We set $\gamma = 0.5$ and let p_0 adjust to match this elasticity. The exogenous separation rate is set to $\delta = 2.6\%$ to match the average transition rate from employment to unemployment in the Current Population Survey (CPS). For an unemployed worker, the probability of having the search opportunity in a period is normalized to $\lambda_u = 1$. A similar probability for an employed worker, λ_e, is chosen to obtain a monthly job-to-job transition rate between 2.2% and 3.2% (e.g., Hornstein et al., 2011). Productivity is normalized to $y = 1$. Home production is set at $b = 0.3$, similar to Hornstein et al. (2011). Finally, the entry cost is set at $k = 0.2$ to match an unemployment rate of 6.5%.

Table 1: Baseline calibration parameters

Parameter	Baseline	Target
β	0.996	annual discount rate = 5%
r	0.367%	annual interest rate = 4.5% and $\beta(1+r) < 1$
δ	2.6%	separation rate into unemployment in CPS
σ	2	standard in macro calibration
γ	0.5	exogenously set
p_0	0.9	elasticity of p_u to θ_u lies in $[0.27, 0.50]$
λ_e	0.23	EE transition rate lies in $[2.2, 3.2]\,\%$
λ_u	1	normalization
y	1	normalization
b	0.3	31% of average wage
k	0.2	unemployment rate = 6.5%

With the identified parameters, we solve the equilibrium and simulate the model for the distribution of individuals (see Appendix B for the procedure). Table 2 shows the model simulated moments and their targets in the data. The model is very close to the data on the first three moments because the calibration uses these moments as targets.[9] The

[8] We examine the effects of changing this bound in Appendix A.

[9] The model was calibrated by numerical exploration. A finer match can be achieved but more exercises are needed. The solution of the model takes about 24 hours to converge, making any loss minimizing

model also yields the average elasticity of p_u with respect to θ_u that is within the values considered as reasonable in the literature.

Table 2. Targeted moments

	Baseline	Data
Unemployment rate	6.49%	6.5%
EU transition rate	2.6%	2.6%
EE transition rate	3.16%	[2.2,3.2]%
Elasticity of p_u to θ_u	0.33	[0.27,0.50]

The computed policy functions will be analyzed in the next subsection and the transition rates in subsection 4.3. Subsection 4.4 will analyze the distribution and inequality.

4.2. Policy functions, value functions and market tightness

A critical feature of the equilibrium is that an individual's decision on consumption and savings interacts with search decision in the labor market. We analyze this interaction using the computed policy functions.

Figure 1 depicts workers' optimal search decisions (the top panel), the ratio of next period's wealth to current wealth (the middle panel) and consumption (the bottom panel), all as functions of current wealth. In each panel, the green dashed line is for an unemployed worker. The other three lines are for employed workers at three levels of current earnings: low $w \in (b, \hat{w}_u(a))$ (the black solid line), medium $w \in (\hat{w}_u(a), \bar{w})$ (the red dashed line), and high $w = \bar{w}$ (the blue line). If the current wage is equal to the maximum, the target wage for search stays constant at this maximum over all wealth levels, as depicted by the blue line in the top panel. Note that the low wage that gives rise to the black solid line is out of the equilibrium, since it is lower than $\hat{w}_u(a)$, the target wage searched by an unemployed worker with the same wealth. Because on-the-job search induces a wage ladder and a worker does not fall down on the ladder, except being hit by the exogenous separation shock, the ergodic distribution of wages does not contain any wage below $\hat{w}_u(a)$. Also note that the low wage is sufficiently low so that a worker employed at such a low

procedure very costly in terms of time.

wage out of the equilibrium would search for a wage lower than $\hat{w}_u(a)$. This is caused by the assumption that an employed worker has less opportunity to search, i.e., $\lambda_e < \lambda_u$.

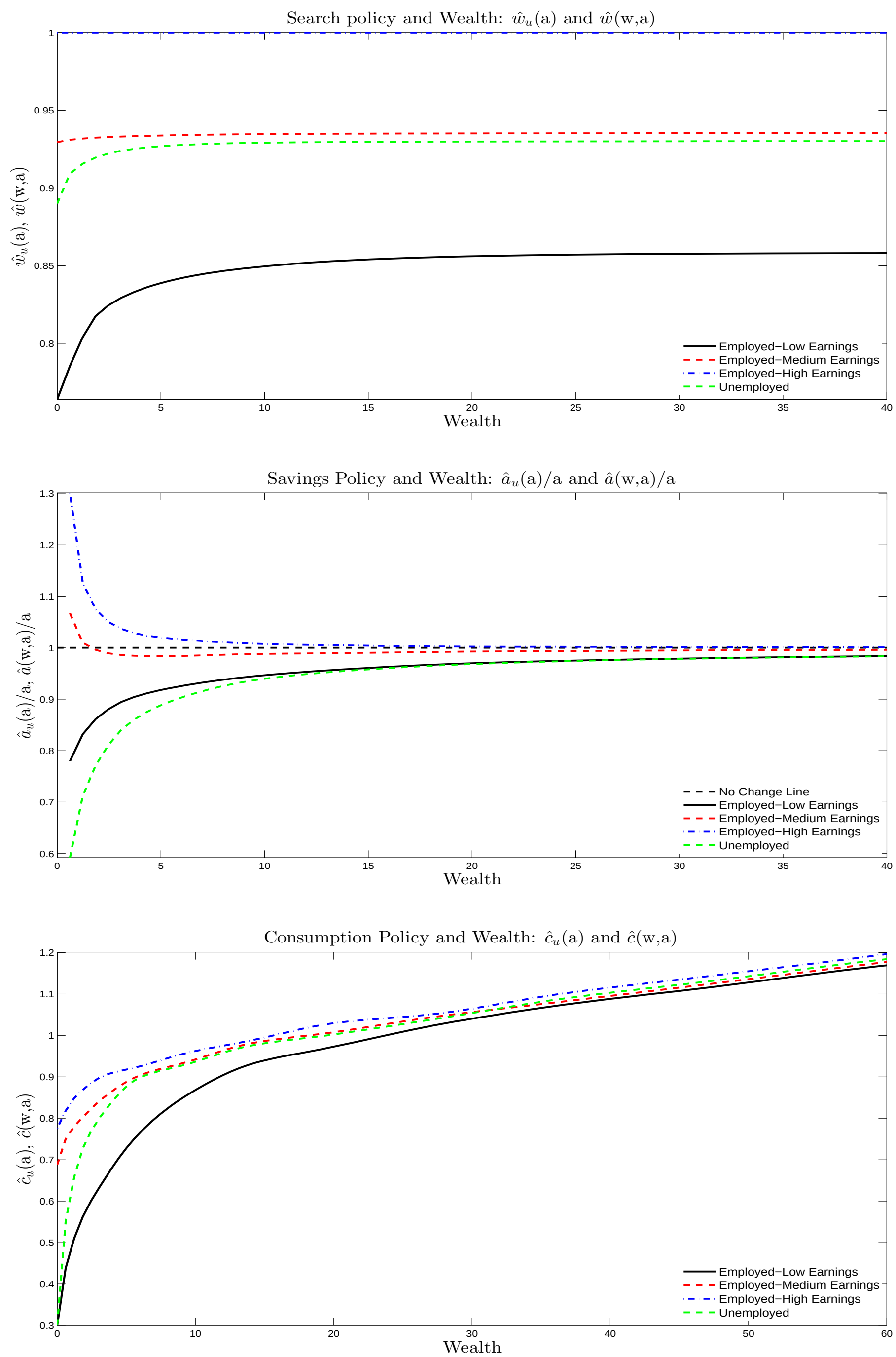

Figure 1. Optimal search target, future wealth and current consumption as functions of wealth for earnings fixed at low $w \in (b, \hat{w}_u(a))$, medium $w \in (\hat{w}_u(a), \bar{w})$, and high $w = \bar{w}$.

The effect of wealth on job search is strong at low wealth levels and dissipates as wealth increases. For all earnings lower than the maximum, the target wage for search is increasing in wealth if wealth is moderate or low. This policy function becomes flat when wealth is high. To explain these patterns, consider a worker with low wealth. Even if the borrowing limit is not currently binding on the worker, it is likely to be binding soon if the wage does not increase, in which case future consumption will fall. To partially self-insure against this outcome, the worker tries to obtain a wage increase quickly by search. The optimal target wage for search is low because only low-wage jobs have high job-finding probabilities. An increase in wealth reduces the likelihood that the borrowing limit will be binding soon. This enables the worker to tolerate a lower job-finding probability and, hence, to search for higher wages. That is, the target wage as a function of wealth is positively sloped at low wealth levels. As wealth keeps increasing, the effect diminishes, and so the target wage policy function becomes less steep. When wealth is sufficiently high, the worker is perfectly self-insured against income risks, in which case further increases in wealth do not affect the optimal target wage for search.

The middle panel in Figure 1 shows that, as wealth increases, the ratio of future to current wealth converges to one; i.e., a worker becomes perfectly self-insured. The path of future wealth depends on current earnings. When a worker's earnings are low, the ratio of future to current wealth is lower than one. In this case, the worker is decumulating wealth in order to maintain smooth consumption. The extent of this decumulation decreases when current wealth rises, and so the ratio of future to current wealth increases toward one. In contrast, for a worker with high earnings, the ratio of future to current wealth is above one. In this case, the worker is accumulating wealth as a precaution for exogenous job separation into unemployment. For a worker with medium earnings, the path of wealth is non-monotonic, as shown by the red dashed line in the middle panel in Figure 1. The worker accumulates wealth first as the motive of precautionary savings dominates, and then decumulates wealth as the consumption smoothing motive dominates. In general, for workers with lower current earnings, the ratio of future wealth to current wealth is lower but increases more quickly than for workers with higher current earnings.

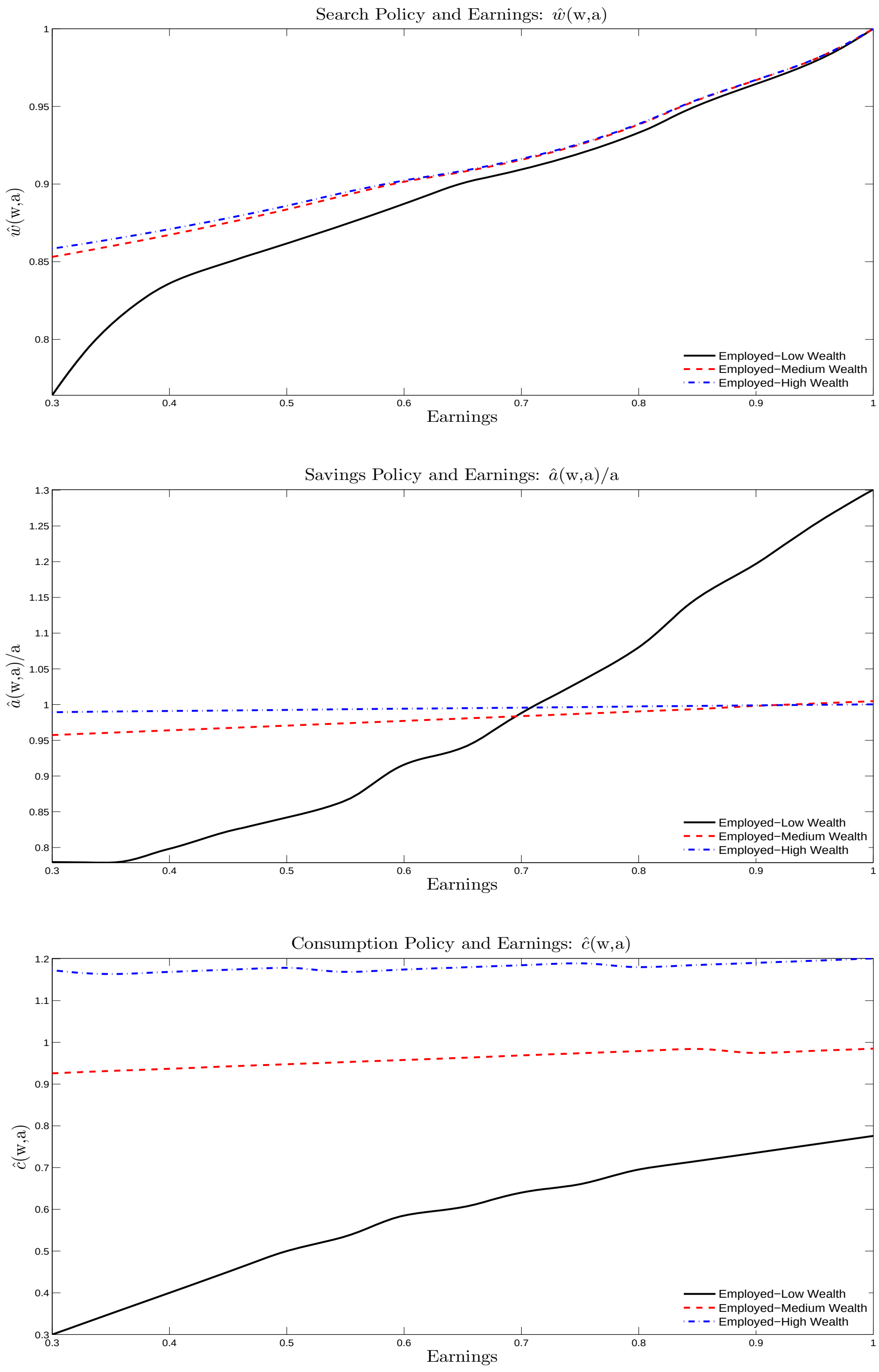

Figure 2. Optimal search target, future wealth and current consumption as functions of earnings for wealth fixed at low, medium and high levels.

The bottom panel in Figure 1 shows consumption as an increasing function of wealth. The slope of the consumption function represents the marginal propensity to consume out of wealth, which decreases as wealth increases to improve the ability to self-insure. When wealth approaches the level of perfect self-insurance, consumption becomes constant, and so the marginal propensity to consume approaches zero. Moreover, for any given wealth, workers with low earnings have lower consumption and, to smooth consumption, these workers consume more out of their wealth proportionally. This effect of current earnings is reflected by the feature that the consumption function is lower but steeper for workers with low earnings than for workers with high earnings.

Figure 2 depicts employed workers' target wage for search (the top panel), the ratio of future to current wealth (the middle panel), and consumption (the bottom panel), all as functions of current earnings. In each panel, the three lines correspond to three wealth levels: low (the black solid line), medium (the red dashed line) and high (the blue dot-dashed line). These panels confirm the above analysis. Relative to workers with high earnings, workers with low earnings search for lower wages, decumulate wealth more to smooth consumption, and have higher propensities to consume out of wealth. These differences narrow as wealth increases. When wealth is high, consumption and the ratio of future to current wealth change little with earnings.

Now we examine how the firm value of a filled job and the market tightness depend on the wage and the applicant's wealth. In Figure 3, the left panels depict the dependence on an applicant's wealth, where the applicant's current earnings are set to three levels with the same legends and colors as in Figure 1. The right panels depict the dependence on the wage offer, where the applicant's wealth is set to be three levels with the same legends and colors as in Figure 2. Not surprisingly, the firm value of a filled job is a decreasing function of the wage offer for any given wealth level of the applicant. Anticipating the low value of a filled job, not many firms enter the submarket to offer the high wage. Thus, the market tightness is also a decreasing function of the wage offer.

For any given wage offer, the firm value of a filled job and the tightness of the submarket offering the wage are increasing functions of an applicant's wealth, provided that the applicant's current wage is lower than the maximum. This positive dependence arises because a worker's wealth affects search decisions as analyzed above. When a worker's wealth is high, it is not urgent for the worker to obtain a wage increase for self insurance. As a result, the worker will search for higher wages that are less likely to be obtained. This reduces the endogenous separability probability of the worker from the current job and, hence, increases the firm value of the job filled by the worker. Anticipating this higher value of a job filled by a wealthier worker, more vacancies enter the submarket to attract such workers. The tightness increases in this submarket. These effects of a worker's wealth weaken as the wage offer increases. When the wage offer is at the maximum, a worker employed at such a wage is not expected to move to another job. In this limit, the firm value of a filled job and the tightness of the submarket offering the maximum wage are independent of the worker's wealth.[10]

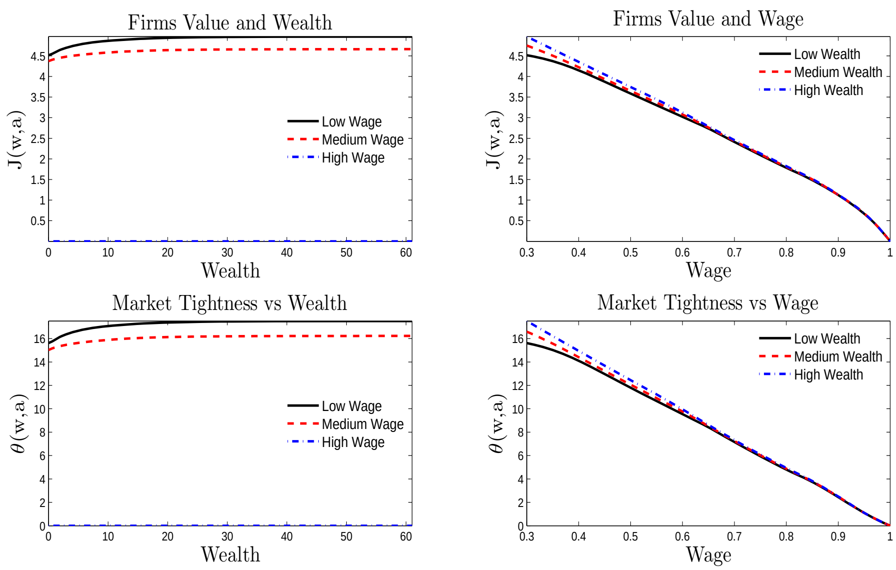

Figure 3. Firm value and market tightness as functions of worker's wealth and the wage offer

Figure 4 summarizes the analysis in this subsection in three dimensional graphs. It plots a worker's value function $V(w,a)$, the return to on-the-job search $R(w,a)$, the value of a filled position $J(w,a)$, and the market tightness function $\theta(w,a)$.

[10]At very low wage and wealth levels, the benefit to a firm of increasing the wage to retain a worker may be even higher than the direct cost of the wage increase. In this case, an increase in the wage offer improves the payoff to both a recruiting firm and an applicant. Thus, submarkets with such low wages are not active in the equilibrium.

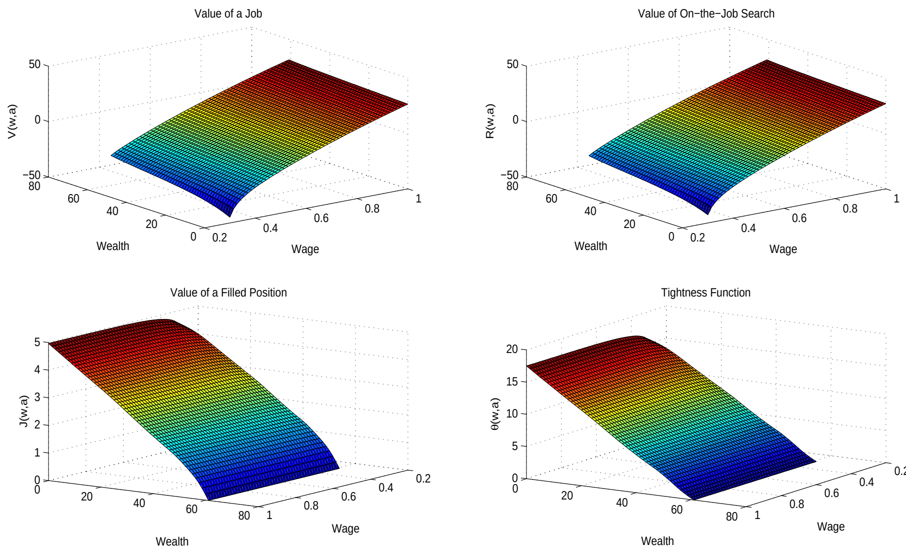

Figure 4. Workers' value function, return on search, firm value, and market tightness as functions of workers' wealth and earnings

4.3. Worker transition

By affecting workers' search decisions, wealth affects the transition rates of workers in the labor market. To gauge the significance of this effect, we use the model simulated data to run the following cross-sectional regressions:

$$U2E_i = \beta_0^{u2e} + \beta_a^{u2e} a_i + \varepsilon_1,$$
$$J2J_i = \beta_0^{j2j} + \beta_a^{j2j} a_i + \beta_w^{j2j} w_i + \varepsilon_2.$$

U2E is the monthly transition rate from unemployment to employment, J2J is the transition rate directly from one job to another job, and the subscript i indicates individuals. To check for robustness, we replace the regressors with log regressors in the above regressions, i.e., a_i with $\ln a_i$ and w_i with $\ln w_i$. Table 3 lists the coefficients of the two types of regressions.

Table 3. Effects of an applicant's wealth and wage on job transition rates

Regressors\Coefficients	β_a^{u2e}	β_a^{j2j}	β_w^{j2j}
Level regressors (a_i, w_i)	-0.0091 [3.79]	-0.0011 [3.85]	-0.5711 [108.56]
Log regressors $(\ln a_i, \ln w_i)$	-0.0152 [3.13]	-0.0016 [2.48]	-0.5477 [108.11]

Note: The numbers in [.] are Newey-West adjusted t statistics.

The two types of regressions yield similar results. All regression coefficients are statistically significant. Wealth affects both the U2E and J2J transition rates negatively. That is, the higher the wealth, the lower the transition rates. This finding is consistent with the analysis in section 4.2. Namely, wealthier individuals apply for better paid jobs and face a lower job-finding probability. Similarly, a worker's current wage negatively affects the transition rate to another job. For any given wealth, the target wage for search increases in a worker's current wage, which comes with a lower job-finding probability.

4.4. Distribution of workers and inequality

Table 4 reports measures of inequality in earnings, income, wealth, and consumption. Income is equal to earnings plus interest payment on assets. The results in the baseline model, reported in the first column, are compared with those in two benchmarks:
(i) The "no-search" model (the second column in Table 4): This is similar to Aiyagari (1994), where the labor market is frictionless but individuals face employment risks and a borrowing limit.[11] In each period, a job is destroyed exogenously with probability δ and the worker is unemployed. An unemployed worker becomes employed with probability $p_u \in (0, 1)$. The probability p_u is set to be equal to the average job-finding probability of an unemployed worker in the baseline model. The wage is the competitive level that sets the profit of a vacancy to zero.
(ii) The "no-wealth" (the third column in Table 4): This is a model where individuals are hand-to-mouth. As in the baseline model, employed and unemployed workers can search, search is directed, and workers are risk averse.

Appendix C describes these two benchmarks in more detail. Note that in the baseline and the no-wealth model, all wage inequality is frictional in the sense that it is caused by search frictions. Since all workers have the same ability and preferences, and all jobs produce the same amount of output, all workers would have the same wage if search frictions were absent in the labor market as in the no-search model.

[11] This benchmark differs from the model in Aiyagari (1994) primarily in that the interest rate is exogenous rather than endogenous.

The mean-min wage ratio, proposed by Hornstein et al. (2011) as a measure of wage inequality, is the ratio of the average wage earned by an employed worker to the lowest wage in the equilibrium. Our model generates a mean-min wage ratio of 1.085. To put this number in a perspective, it is useful to know that the mean-min wage ratio is no more than 1.04 in a variety of search models, as demonstrated by Hornstein et al. (2011).[12]

Table 4. Inequality measures (not targeted)

	Baseline	No-search	No-wealth
Mean-min wage ratio	1.085	1.000	1.062
Gini: wealth	0.262	0.310	——
Gini: earnings	0.058	0.045	0.067
Gini: income	0.063	0.055	0.067
Gini: consumption	0.027	0.026	0.067
$\frac{\text{Gini: consumption}}{\text{Gini: earnings}}$	0.466	0.518	1.000

The main cause of the higher wage inequality in our model is the interaction between wealth accumulation and search. Ignoring wealth accumulation, as in the no-wealth model, results in lower frictional wage dispersion. As explained in section 4.2, the desire for self insurance motivates unemployed workers to take jobs that pay low wages. This expands the left tail of the equilibrium wage distribution. As wealth increases, a worker becomes better insured and can take the chance of searching for higher wages. This expands the right tail of the wage distribution. Despite these forces that can increase wage inequality, the mean-min ratio in our model is only slightly higher than in the no-wealth model. It is worth noting that Krusell et al. (2010) also analyze search with wealth accumulation but they do not allow for on-the-job search. Their model generates a mean-min ratio of 1.02 when it is calibrated as in Shimer (2005).[13] The presence of on-the-job search in our model is important for the higher dispersion, as examined further in Appendix A.

Table 4 also reports Gini coefficients in wealth, earnings, income, and consumption. The baseline model shows more earnings and income inequality than the no-search model. In the latter model, there is no dispersion in wages, and so earnings inequality is only

[12]Shi (2016) constructs a directed model of on-the-job search that can generate a mean-min ratio of 1.81. In his model, the cost of posting a vacancy is sufficiently convex in the vacancy type. In the equilibrium, firms create low starting jobs for unemployed workers first and upgrade jobs later.

[13]They also get a mean-min ratio of 1.0002 when it is calibrated as in Hagedorn and Manovskii (2008).

driven by the uncertainty in whether workers are employed. However, notice that the ratio of the Gini in consumption to the Gini in earnings is lower in the baseline model than in the no-search model. This ratio can be interpreted as the proportion of earnings risks that are passed through to consumption. Thus, individuals in the baseline model are able to shield consumption better from fluctuations in earnings than in the no-search model. This better insurance comes from individuals' ability to choose which jobs to apply for as well as the amount of savings. Moreover, if individuals are hand-to-mouth, as in the no-wealth model, then all earnings risks are passed through to consumption.

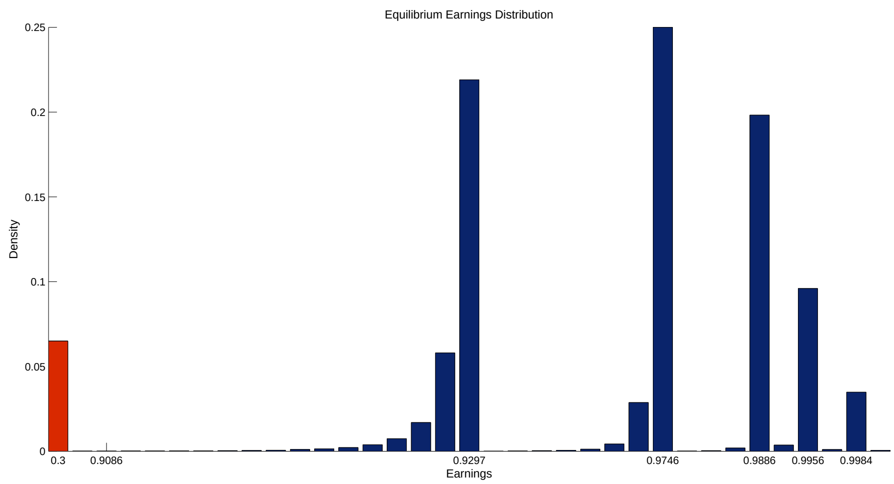

Figure 5. Equilibrium earnings distribution

It is informative to go beyond the statistics of inequality to examine the entire distributions of wages and wealth. Figure 5 shows the equilibrium density of wages. The support of the distribution is endogenous and discrete. The red bar at earnings equal to 0.3 shows the mass of individuals who are unemployed. The blue bars show the mass of individuals earning each of the equilibrium wages, which range from 0.89 to almost 1. Most employed workers are employed at a few values of wages. The shorter bars to the left of the tall bars are the densities of workers who have low wealth and apply for lower paid wages in order to have higher matching probabilities. Those workers are the ones for whom the wealth effect on job search decision is stronger.

Before showing the wealth distribution, we depict the savings policy function for all levels of equilibrium earnings and wealth. In Figure 6, the green dashed line shows the savings policy of unemployed workers, expressed as $\hat{a}_u(a) - a$. The blue shaded area represents savings policies of employed workers at all equilibrium wages, expressed as $\hat{a}(w, a) - a$. Unemployed workers decumulate wealth at all wealth levels, since $\hat{a}_u(a) - a < 0$ for all a. They do so to smooth consumption under the expectation of finding a job and getting an increase in income in the future. In contrast, a large part of the blue shaded area in Figure 6 lies above zero, which means that many employed workers accumulate wealth. The motive for savings comes from the precaution for exogenous separation into unemployment. In the equilibrium, even the lowest wage of an employed worker is much higher than home production in unemployment. Losing such high earnings represents a large risk which an employed worker wants to insure against by savings. This motive of precautionary savings is particularly strong when an employed worker has low wealth. Even when wealth is high, an employed worker still accumulates wealth if earnings are high. An employed worker decumulates wealth only when wealth is high and earnings are low. In this case, the motive of reducing wealth to smooth consumption dominates the motive of precautionary savings.

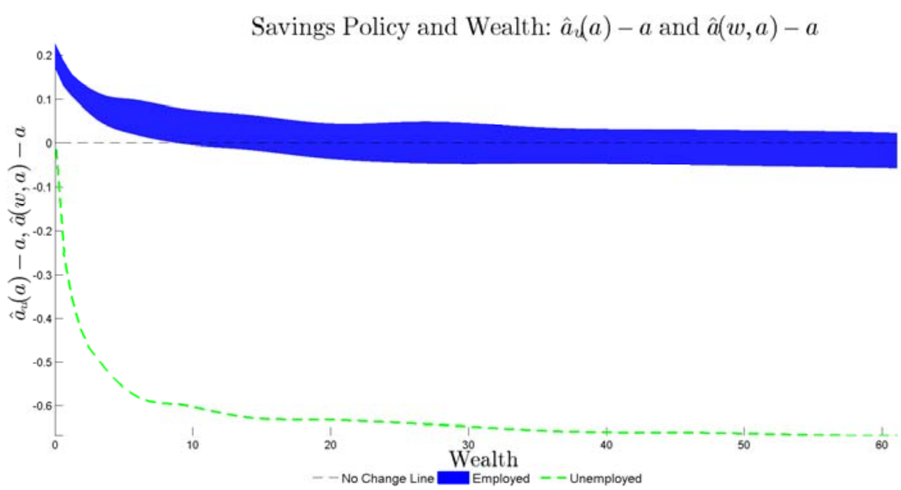

Figure 6. Equilibrium saving policy

Table 5 shows the fraction of individuals that are accumulating and decumulating wealth according to their position with respect to the average asset holdings in equilibrium ($\overline{a} = 14.94$). The elements on the main diagonal of the table show the fraction of the population that push the wealth distribution towards the mean. The two elements off the diagonal show the fraction of the individuals that are spreading the wealth distribution away from the mean. In total, 36.4% of the population is spreading wealth away from the mean, with around $\frac{4}{5}$ of them pushing the right tail of the distribution. In addition, 63.6% of the population is contracting the distribution towards the mean.

Table 5. Fraction of workers accumulating and decumulating wealth

current wealth \ future wealth	$\hat{a} \leq a$	$\hat{a} > a$
$a > \bar{a}$	13.1%	29.1%
$a \leq \bar{a}$	7.3%	50.5%

The above behavior of savings directly affects the wealth distribution in equilibrium. Figure 7 plots the Lorenz curve, and Figure 8 plots the density function of the wealth distribution. The model is able to capture some of the qualitative features of the U.S. distribution of wealth, such as positive skewness, a short and fat left tail, and a long right tail (see Budria et al., 2002). Figure 8 also shows how the wealth distribution in the baseline model compares to the no-search model. The baseline model is able to generate a longer right tail and larger mass of wealthy individuals. This result is interesting since many models have difficulty to generate a long right tail of the wealth distribution.

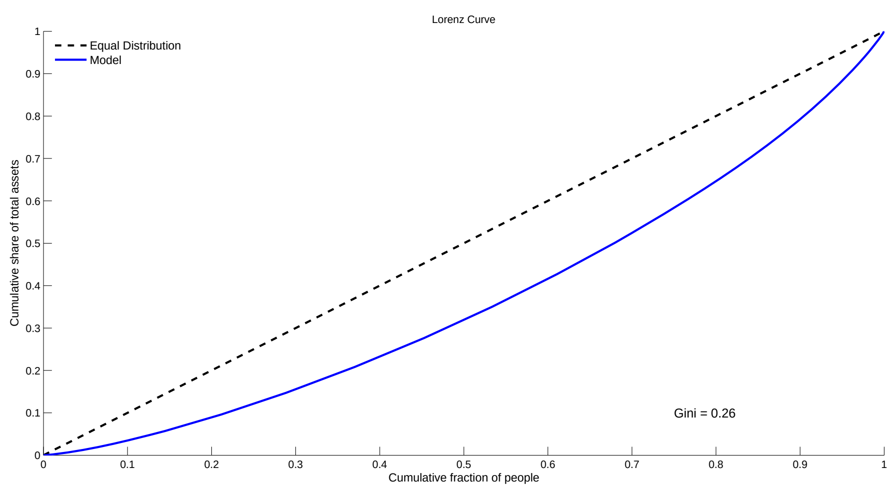

Figure 7. Lorenz curve of wealth

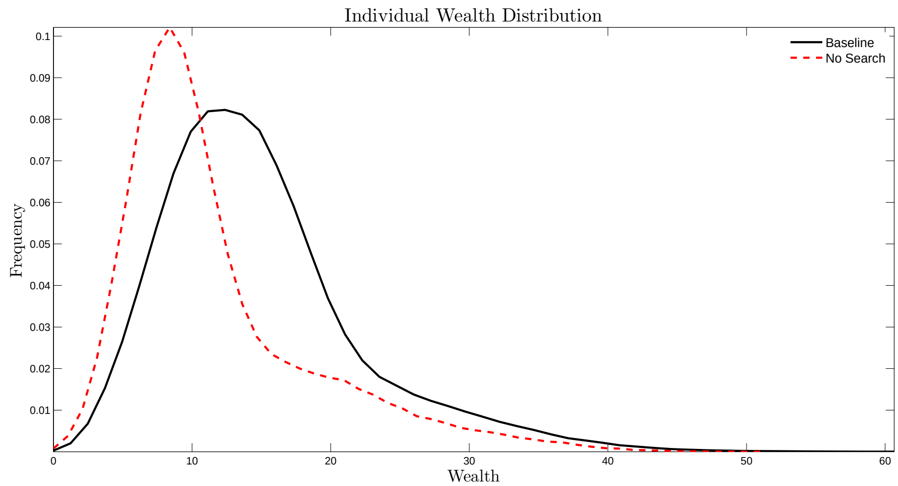

Figure 8. Wealth distribution

In Appendix A we conduct counter-factual exercises to examine the roles of on-the-job search, the borrowing limit and the interest rate. The results can be summarized as follows. First, shutting down on-the-job search reduces the mean-min wage ratio from 1.085 to 1.023. On-the-job search is important for wage dispersion by reducing the minimum wage in the equilibrium. It also generates a longer right tail and less concentration around the mean in the wealth distribution. Second, relaxing the borrowing constraint (i.e., reducing $\underline{a}$) does not change wage dispersion much, but it makes the left tail of the wealth distribution decompress. Thus, a tight borrowing limit is important for generating a short and fat left tail in the wealth distribution. Surprisingly, relaxing the borrowing constraint *increases* consumption inequality, because it increases the mass of individuals who have large debt and spend part of income on debt repayment. Finally, a lower interest rate reduces the incentive to save and reduces wealth inequality. As individuals are less self-insured by asset holdings, consumption inequality increases. Similarly, the lower is the interest rate, the lower is the skewness and the shorter is the right tail of the wealth distribution. To generate a shape of the wealth distribution similar to the one observed in the data, the interest rate must be close enough to the upper limit, $\frac{1}{\beta} - 1$.

5. Conclusions

Using an equilibrium search model with wealth accumulation, we have focused on (i) how much wage and wealth dispersion can be generated by a model of directed on-the-job search, and (ii) how wealth affects job search decisions. The results in the baseline model have been compared with two benchmark models studied in the literature, a "no-search" model and a "no-wealth" model. We have found that wealth significantly reduces a worker's transition rates in the labor market. Moreover, search frictions increase wealth inequality significantly. Relative to the no-search model with employment risks, the baseline model is able to better reproduce some salient features of the U.S. wealth distribution, such as a positive skewness, a short and fat left tail, and a long right tail. In particular, the baseline model produces a larger mass of wealthy individuals than the no-search model. By comparing the baseline model with the no-wealth model, we have found that the effect of wealth on search increases wage dispersion, but only by a small amount. Frictional wage dispersion is small relative to the data once the model is disciplined to match the high job-finding probability of unemployed workers in the data. Frictional wage inequality becomes even smaller if workers are not allowed to search on the job.

Several extensions of this model are worth pursuing. First, firms can post dynamic contracts instead of a fixed wage. With dynamic contracts, firms have incentive to back-load wages to increase retention, as analyzed by Burdett and Coles (2003) and Shi (2009). This force can stretch the upper tail of the wage distribution. Also, an unemployed with low wealth may be willing to accept even lower wages than in the baseline model in the expectation of wage increases in a contract. This force can stretch the lower tail of the wage distribution. Although both forces can widen frictional wage dispersion, their quantitative importance is yet to be determined. Moreover, shocks to match-specific productivity and/or work effort can be introduced as in Tsuyuhara (2016) and Lamadon (2016).

Second, the interest rate can be endogenized. In one of the counter-factual exercises, we have examined how the interest rate affects the equilibrium. The shape of the wealth distribution and, especially, the skewness is sensitive to the interest rate. In order to get a reasonably skewed wealth distribution, the interest rate must be close to the upper bound, which is the rate of time preference. It is unclear whether the equilibrium interest rate can be close to this upper bound.

Finally, extending the model to study the business cycle seems a natural exercise. When aggregate shocks are present, individuals may have further incentives to accumulate assets to smooth consumption. The computational advantage provided by block recursivity of the equilibrium makes it tractable to study business cycles.

Appendix

A. Counter-factual Exercises

In this appendix we conduct three counter-factual exercises in turn. First, we shut down on-the-job search (OJS) by setting $\lambda_e = 0$ to show that on-the-job search is important for frictional wage dispersion. Second, we relax the borrowing constraint by changing the borrowing limit from $\underline{a} = 0$ to a negative number. Third, we show that most results in the baseline model are robust to moderate changes in the interest rate. In each exercise, we recalibrate the model to match the targets in Table 1, provided that they remain valid. The main change is in the vacancy cost k. The recalibrated value of k and other results of the counter-factual exercises are reported in Table 6.

Table 6. Results of counter-factual exercises

	Baseline	No OJS ($\lambda_e = 0$)	$\underline{a} = -11$	$r = 0\%$	Data
Vacancy cost k	0.20	0.27	0.20	0.26	——
Unemployment rate	6.49%	6.61%	6.49%	6.52%	6.5%
UE transition rate	2.6%	2.6%	2.6%	2.6%	2.6%
EE transition rate	3.16%	0%	3.16%	3.15%	2.2-3.2%
Elasticity of p_u to θ_u	0.33	0.35%	0.33	0.34	0.27-0.50
Mean-min wage ratio	1.085	1.023	1.091	1.083	1.7-2.0
Gini: wealth	0.26	0.26	0.32	0.20	0.80
Gini: earnings	0.06	0.05	0.06	0.06	0.61
Gini: income	0.06	0.05	0.07	0.06	0.55
Gini: consumption	0.027	0.024	0.033	0.034	0.25
Corr(earnings income)	0.99	0.99	0.98	1.00	0.72
Corr(earnings, wealth)	0.06	0.04	0.04	0.27	0.47
Corr(income, wealth)	0.21	0.18	0.26	0.27	0.60

When on-the-job search (OJS) is shut down, the mean-min wage ratio falls significantly from 1.085 to 1.023. This result shows that most of the frictional wage dispersion in the baseline model comes from on-the-job search. The ratio with no OJS, 1.023, is comparable to the finding in Krusell et al. (2010). Their model allows for savings but not for on-the-job search. When their model is calibrated as in Shimer (2005), the mean-min wage ratio is 1.02. On-the-job search widens wage dispersion mainly by reducing wages that unemployed workers choose to search for. Unemployed workers are willing to lower their search target in the expectation that they can search for higher wages after being employed.

Despite this effect of on-the-job search on the lowest wage, wage dispersion is small once the model is calibrated to match unemployed workers' job-finding probability and the value of home production. As a result, wage dispersion is not the most important force driving the Gini coefficients. Rather, the driving force is the distance between the minimum wage and home production. This distance is similar regardless of whether employed workers can search on the job. Figure 9 shows the density functions of wealth in the baseline model, the model with no OJS and the model with no search. Relative to the baseline, the wealth distribution with no OJS is more concentrated around its peak.

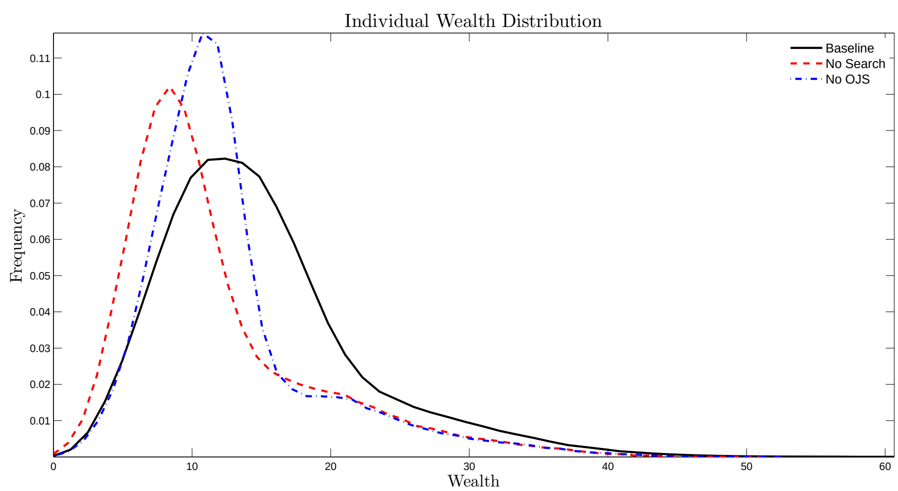

Figure 9. Wealth distribution: baseline vs. no on-the-job search

To examine the role of the borrowing limit, we change the lower bound on wealth from $\underline{a} = 0$ to -11, -22, and -55. These increased borrowing limits are equal to earnings over 1, 2, and 5 years for an individual whose wage is equal to the average wage in the baseline model. To economize on space, Table 6 lists only the change from $\underline{a} = 0$ to $\underline{a} = -11$. Increasing the borrowing limit increases wealth inequality, as indicated by the larger Gini coefficient in wealth. It is notable that consumption dispersion also increases. This result seems puzzling since relaxing the borrowing limit enables individuals to smooth consumption better, which should reduce consumption dispersion. However, the result can be explained since it is akin to the immiserizing effect of debt. That is, relaxing the borrowing limit increases the fraction of the population who end up with negative wealth in the steady state. Since these individuals spend part of their earnings to pay interest on debt, inequality in income increases. So does inequality in wealth.

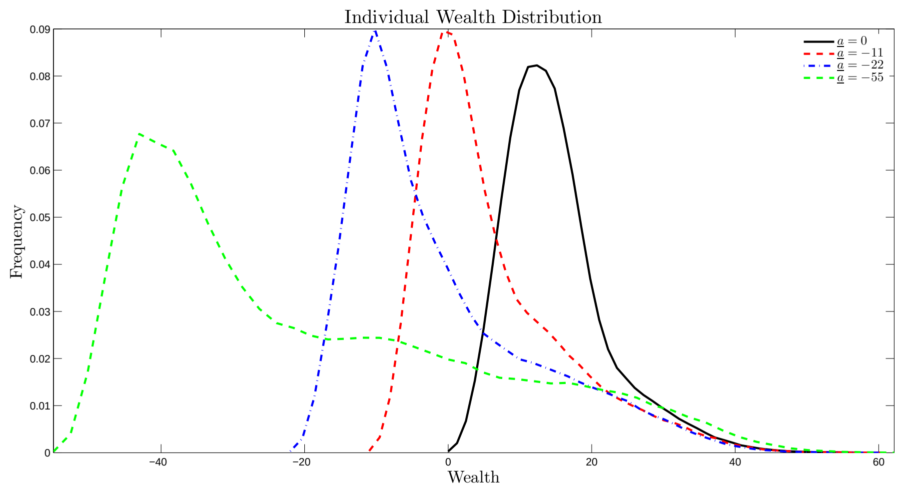

Figure 10. Effects of extending the ad-hoc borrowing limit on the wealth distribution

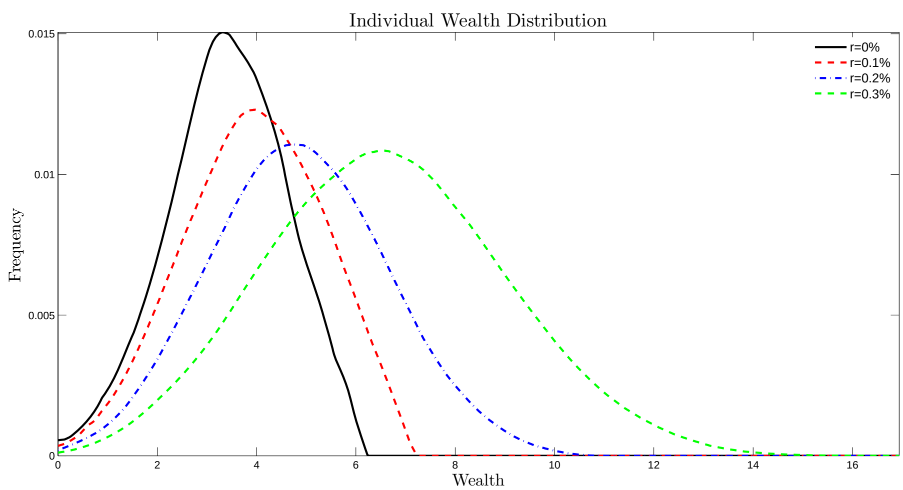

Figure 11. Effects of interest rates on the wealth distribution

Figure 10 shows the density functions of wealth for all four borrowing limits: $\underline{a} = 0$ (baseline), $-11, -22, -55$. Under all four limits, there are some individuals who want to take as much debt as possible. The larger the borrowing limit (larger $|\underline{a}|$), the less concentrated is the distribution around the mean. Also, the larger the borrowing limit, the less positively skewed is the wealth distribution.

Finally, we examine the effect of reducing the monthly interest rate from the baseline value $r = 0.367\%$ to 0%, 0.1%, 0.2%, and 0.3%, in turn. To economize on space, Table 6 lists only the change from the baseline to $r = 0\%$. Reducing the interest rate does not affect wage dispersion significantly. Also, the lower rate of return on assets reduces the incentive for individuals to accumulate assets. As asset holdings fall, consumption is exposed to higher volatility, as shown by the increase in the Gini coefficient in consumption. Figure 11 depicts the density functions of wealth for four values of the interest rate: $r = 0\%, 0.1\%, 0.2\%$ and 0.3%. As the interest rate increases, the wealth distribution expands to the right, showing a longer right tail and peaking at higher wealth levels. This confirms the effect that a higher interest rate increases the incentive to accumulate assets. However, even at $r = 0.3\%$, the skewness of the wealth distribution is low and the right tail is short. To generate a shape of the wealth distribution similar to the one observed in the data, the interest rate must be close enough to the rate of time preference, $\frac{1}{\beta} - 1$.

B. Solution Algorithm and Simulation

We solve the model using a nested fixed point algorithm. First, we define the grids for the spaces of asset holdings and wages. The space of asset holdings is set to be $[\underline{a}, \bar{a}] = [0, 75]$, which is larger than the equilibrium support of the wealth distribution. The space of wages is bounded between the home production level b and the total flow of productions per period y. In equilibrium, there will be no wages outside those bounds. The steps of the algorithm are as follows:

(1) Guess initial value functions of workers and firms.

(2) Given the value function of firms, solve for the market tightness that is consistent with competitive entry of firms.

(3) Given the tightness function, solve a worker's optimization problem and compute optimal savings, consumption, and job search decisions. This step is done by iterating on

the worker's value function until convergence, using the shape-preserving Cubic Hermite interpolation to calculate the policy functions.

(4) Given workers' policy function computed in (3), calculate separation rates of employed workers and update the value function of the firm in each submarket.

(5) Iterate on the value function of the firm until convergence.

Once the value and policy functions are solved, we simulate the model to obtain the distribution of individuals, using $N = 100,000$ individuals and $T = 1,100$ time periods (months). We discard the first 100 periods to avoid dependence on initial conditions. The average of the last 100 periods of the simulations is used to calculate the stationary distribution.[14] To simulate the evolution of individuals' states, we take random draws for separation shocks, the search opportunity (consistent with λ_e and λ_u), and matching shocks that determine which individuals in each submarket are matched. Also, we start the economy by assigning a random state to each of the N workers. Then, we use the equilibrium optimal policy functions and these random shocks to compute the endogenous evolution of the state of each individual.

C. Benchmark Models

In this appendix we present two benchmark models to compare with the baseline model in section 2. The first benchmark is a no-search model where search frictions do not exist in the labor market but individuals are still exposed to employment uncertainty and a borrowing limit. This model is similar to the model in Aiyagari (1994). The second benchmark is a no-wealth model where individuals are hand-to-mouth and search frictions exist in the labor market. This model is the standard model of directed search augmented with on-the-job search and risk aversion.

[14]The distribution appears stationary at most after period 400 in all of the exercises that we have conducted. So, choosing 100 periods is not relevant for the results presented in this paper. We could use only the distribution in the last period and get about the same numbers. However, using more periods allows us to reduce errors.

C.1. No-search model

In this benchmark model, the employment status of a worker is exogenously determined by shocks. In each period, an unemployed worker is hit by an *iid* employment shock. With probability $p_u \in (0,1)$, the worker becomes employed instantly. With probability $1 - p_u$, the worker remains unemployed. The probability p_u is set to be equal to the average job-finding probability of an unemployed worker in the baseline model. As in the baseline model, a job is hit by an *iid* separation shock with probability $\delta \in (0,1)$, in which case the worker becomes unemployed. All employed workers earn the wage that sets the profit of a vacancy to zero. There is competitive entry of vacancies, and the vacancy cost is k per period. Individuals face a borrowing limit. The timing of events in a period is as follows:

(i) Production: Workers start their period with a state (e, a). Production is given by y.

(ii) Consumption and saving: Individuals decide how much to consume and how much to save for the next period.

(iii) Separation and hiring: The separation shock is realized for each match and the employment shock is realized for each unemployed worker.

The value function of an unemployed worker with assets $a \in A$ satisfies:

$$\begin{aligned} V(e_0, a) &= \max_{(c,\hat{a})} \{u(c) + \beta [p_u V(e_1, \hat{a}) + (1 - p_u) V(e_0, \hat{a})]\} \\ s.t. \quad & c + \tfrac{\hat{a}}{1+r} = b + a \quad \text{and} \quad \hat{a} \geq \underline{a}. \end{aligned}$$

The value function of an employed worker with assets $a \in A$, satisfies:

$$\begin{aligned} V(e_1, a) &= \max_{(c,\hat{a})} \{u(c) + \beta [\delta V(e_0, a) + (1 - \delta) V(e_1, a)]\} \\ s.t. \quad & c + \tfrac{\hat{a}}{1+r} = w + a \quad \text{and} \quad \hat{a} \geq \underline{a}. \end{aligned}$$

The firm value of hiring a worker at wage w is

$$J(w) = y - w + (1 - \delta) \beta J(w).$$

Competitive entry of vacancies requires $\beta J(w) = k$ for all w such that $J(w) \geq k$. Solving $J(w)$ from the above equation, we can determine the competitive wage as

$$w = y - k[\frac{1}{\beta} - 1 + \delta].$$

An equilibrium can be defined by adapting the definition in section 3.1. In particular, the market tightness function is irrelevant and the wage rate is given above.

C.2. No-wealth model

In this benchmark, there are search frictions, workers can search off and on the job, but they cannot accumulate wealth. Search is directed and there is competitive entry of vacancies into submarkets. The timing of events in a period is the same as in the baseline model. The value function of an unemployed worker satisfies:

$$V_u = u(b) + \beta V_u + \beta \max_{\hat{w}} \lambda_u p(\theta(\hat{w}))\left[V(\hat{w}) - V_u\right].$$

The value function of an employed worker with wage $w \in W$ satisfies:

$$\begin{aligned} V(w) = \; & u(w) + \beta\left[\delta V_u + (1-\delta) V(w)\right] \\ & + \beta \max_{\hat{w}} \lambda_e p(\theta(\hat{w}))\left[V(\hat{w}) - \delta V_u - (1-\delta) V(w)\right]. \end{aligned}$$

The value for a firm of hiring a worker in submarket $w \in W$ is given by

$$J(w) = y - w + (1-\delta)\left[1 - \lambda_e p(\theta(\hat{w}))\right] \beta J(w).$$

By free entry, in any given submarket $\hat{w} \in W$ it must hold that

$$q(\theta(\hat{w}))\, \beta J(\hat{w}) \leq k \text{ and } \theta(\hat{w}) \geq 0,\ \forall \hat{w},$$

where the two inequalities hold with complementary slackness.

An equilibrium can be defined by adapting the definition in section 3.1. In particular, the wealth level is set to 0.

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