Churning, Firm Inter-connectivity, and Labor Market Fluctuations

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Abstract

In the US, during downturns of economic activity, firms change their profit rankings more often. Motivated by this fact, this paper studies the effect of firms' transitions in profit distributions, or churning, on the business cycle through the lens of a Diamond-Mortenson-Pissarides search and matching model. The main prediction of the model is that an increase in the churning of an industry causes a recession within the industry and in its upstream and downstream industries, which is consistent with the evidence I document in the paper. The model's key mechanism, furthermore, is supported by microeconomic evidence.

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In the US, periods of economic downturn are also periods of accelerated movement of firms' rankings across the profit distribution. That is, during the decline of economic activity, there is an increase in the churning of firms' rankings in the profit distribution. Moreover, a higher churning of an industry is usually accompanied by a slower employment growth in its upstream and downstream industries. Motivated by these facts, I study the effect of churning on the labor market, and how the effect spills over the supply chain, through the lens of a Diamond-Mortenson-Pissarides (DMP hereafter) model. Based on my model, I argue that variation in churning at the industry level is a significant factor explaining the cyclical aggregate labor market fluctuations, and firm inter-connectivity across industries serves as a crucial propagation mechanism.

The main idea of the paper is that firms collaborate with partners from the other industries. In choosing partners, firms care about the quality of their partners. The churning of firms' rankings increases the chance that they will match with partners who they are reluctant to work with. When the termination of an existing match is costly, both the firms and their potential partners would optimally avoid inter-firm cooperation which depresses other economic activities such as recruiting of workers.

In section 2, I implement the idea by supplementing the canonical DMP model with firm inter-connectivity. I assume that all firms prefer to cooperate with high-ranking partners; hence, in equilibrium, firms only initiate partnership with similarly ranked partners. I show that churning increases unemployment if and only if the production function is supermodular; that is, firms have a comparative advantage in cooperating with a similarly ranked partner. The intuition is that when the production function is supermodular, cooperation between similarly ranked firms maximizes their aggregate profit. Churning generates mismatch between differently ranked firms, which decreases firms' expected profits and reduces their incentive to create jobs. The main prediction of the model is that an increase in churning of one industry would cause an aggregate recession.

In section 3, I examine a case in which the firms can take action to hedge against the risk of churning. Specifically, I let firms choose the duration of inter-firm cooperation contract. In the model, when a firm is uncertain about its future type or its partner's future type, and if mismatches are inefficient, the firm does not want to make the commitment by choosing a long-term contract. Therefore my model predicts that with supermodularity, fewer firms choose long-term contracts, such as vertical integration, in industries with a higher churning or whose upstream and downstream industries have a higher churning. I find that the prediction is consistent with the evidence in the data, which supports the hypothesis of supermodularity.

The question of the extent to which churning contributes to the business cycle, however, remains to be seen. In section 4, I answer this question by embedding the simple model into a real business cycle (RBC) model disturbed by shocks to churning and several other shocks that have been commonly studied in dynamic stochastic general equilibrium (DSGE) models. After estimating the model using Bayesian method, I find that shocks to churning emerge as the major source of persistent joint movements in unemployment and other macro variables: they account for 27 percent of variation in unemployment and 32 percent of variation in aggregate output.

As a byproduct, my paper provides a theory of endogenous total factor productivity (TFP) and speaks to the Shimer puzzle (Shimer (2005)). According to my estimates, the production function is supermodular, which implies that mismatched firms are, on average, less productive. Churning of an industry raises the share of mismatched firms in the existing matches both within the industry and in its linked industries, leading to a gradual decline in aggregate TFP. This decline in aggregate TFP, however, is mild compared with the surge in unemployment.

My paper is related to the literature on uncertainty shock. Uncertainty shock, as defined by Bloom (2009) and Bloom et al. (2012), characterizes the exogenous change in volatility of idiosyncratic productivity. In many settings, uncertainty shocks induce fluctuations in the rate of churning by varying firms' transition probability in the productivity distribution. This paper is the first to study the spill-over effect of uncertainty shocks.[1] In particular, my paper has a new and attractive implication that industry uncertainty shocks cause economic downturn not only within the industry but also in the upstream and downstream industries, which is consistent with the evidence in the data. Also, most research in this literature emphasizes that firms concerns with the uncertainty of their business condition due to the irreversibility and the non-convex adjustment cost of capital and labor input. The innovation of my paper is to argue that firms also concern with the uncertainty about the business condition of their partner due to the irreversibility of partnership and the opportunity cost of mismatch. Lastly, as shown by Schaal (2012), the standard DMP model predicts an increase in the job creation in response to a higher uncertainty, which is inconsistent with data. In contrast, my model implies that a high uncertainty generates a large decline in the job creation.

[1] While no existing work studies the interaction between industry linkage and uncertainty shocks, Alessandria et al. (2015) show that in a two-country trade model, uncertainty shocks to one country induce negative comovement of the two countries via the non-convex adjustment cost channel.

1 Empirical evidence

This section documents the empirical evidence that motivates my research. I first construct measures of churning at three digit NAICS industry level in the US economy. Then I present several empirical facts regarding the relationship between churning and the economic condition. Particularly, I show that an industry's churning is negatively correlated with the employment growth in the industry's upstream and downstream industries.

1.1 Measuring churning of firms' profitability rankings

I use compustat fundamentals annual from 1960 to 2013. Compustat fundamentals annual is a data set of listed companies which contains more than 370,000 observations and covers 112 3-digit NAICS industries. For each period and within each industry I rank firms by profitability,[2] which is measured by the ratio of profit (Earnings Before Interest, Taxes, Depreciation and Amortization (EBITDA)) to sales. Then I categorize firms into two types, high (H) and low (L), based on ranking; a firm is H type if its profitability measure is above median, L type if below median.

A rotation occurs when a firm changes its type in consecutive periods, either from H to L or vice versa. Industry rotation rate is the fraction of firms that have changed type. Specifically, industry rotation rate for industry i in period t is defined as

$$Rot_{i,t} = \frac{\#rotation_{i,t}}{\#firm_{i,t}}$$

Industry rotation rate $Rot_{i,t}$ measures the churning of firms' rankings in industry i in period t. A high industry rotation rate implies that a firm is more likely to change its ranking within the industry profitability distribution.

It is worth noting that there exist several alternative ways to measure churning, such as the autocorrelation of firms' ranking vector. I choose rotation rate because it is the exact empirical counterpart of churning in the model described in the following sections. In this way, the rotation rate constructed in this section can be used as observable in the estimation of the DSGE model.

[2]Ideally, firms should be ranked by their "true abilities" to promote the inter-firm match's joint pay-off, which is difficult to measure. Yet it is still reasonable to conjecture that profitability is positively correlated with the "true abilities". Several studies, such as Rhodes-Kropf and Robinson (2008), found that high-earning firms are more likely to match with high-earning firms in M&A activities, which justifies profitability as a legitimate proxy.

To measure the churning in an industry's upstream industries, I construct the weighted average of supplier industries' rotation rate

$$Rot_{i,t}^{upstream} = \sum_j w_{i,j}^{upstream} Rot_{j,t}$$

, where $w_{i,j}^{upstream}$ is the the ratio of industry i's intermediate input from industry j to its total intermediate input.

Similarly, I use the weighted average of customer industries' rotation rate to measure the churning in an industry's downstream industries

$$Rot_{i,t}^{downstream} = \sum_j w_{i,j}^{downstream} Rot_{j,t}$$

, where $w_{i,j}^{downstream}$ is the the ratio of industry i's intermediate output to industry j to its total intermediate output.

To differentiate the very sizable change in profitability ranking from the small variation in profitability around the median point, I refine the categorization of firms into four quartiles. By definition, rotation is the switching of rankings from 1st and 2nd quartiles to 3rd and 4th, or vice versa. The switchings between 2nd and 3rd quartile might contain very small variations of profit around the median point. One way to control for the very small variation of profit is to ignore switchings between 2nd and 3rd quartiles and focus only on the ones between non-adjacent quartiles, which I denote as large rotations as they indicate large swings within the profitability distribution.

1.2 Panel regression: employment growth and churning

I use the following regressions to quantify the relationship between employment growth of an industry and churning of the supplier and customer industries

$$Employment\ growth_{i,t} = X_i + \gamma_t + \beta_1 \cdot Rot_{i,t} + \beta_2 \cdot Rot_{i,t}^{upstream} + \varepsilon_{i,t}$$
$$Employment\ growth_{i,t} = X_i + \gamma_t + \beta_1 \cdot Rot_{i,t} + \beta_2 \cdot Rot_{i,t}^{downstream} + \varepsilon_{i,t}$$

Industry employment growth rate is the the dependent variable. For the independent variables, I include industry fixed effect X_i and year effect γ_t.$Rot_{i,t}^{upstream}$ and $Rot_{i,t}^{downstream}$ are measures of churning for industry i's upstream and downstream industries. I also control

Table 1: Measure of churning in the upstream/downstream are negatively correlated with industry employment growth

	Upstream		Downstream	
Measures of churning	Rotation rate	large rotation rate	Rotation rate	large rotation rate
	(1)	(2)	(3)	(4)
$Rot_{i,t}$	-0.09^{***} (0.03)	-0.18^{***} (0.05)	-0.09^{***} (0.02)	-0.18^{***} (0.04)
$Rot_{i,t}^{upstream}$	-0.20^{***} (0.05)	-0.33^{***} (0.09)		
$Rot_{i,t}^{downstream}$			-0.13^{**} (0.06)	-0.22^{**} (0.10)
$\#\, of\ observations_{year\times industry}$	16×42	16×42	16×42	16×42

$eg_{i,t}$ = industry employment growth, NIPA, 1999-2014
X_i = industry fixed effect, γ_t = year effect
$Rot_{i,t}$ = industry rotation rate in year t, Compustat
$Rot_{i,t}^{upstream}$ =rotation rate in the upstream industries in year t, Compustat
$Rot_{i,t}^{downstream}$ =rotation rate in the upstream industries in year t, Compustat
The analysis includes private non-farm industries that are in both Compustat and NIPA and have more than 8 firms in Compustat from 1999 to 2014
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.

for industry i's measure of churning $Rot_{i,t}$ to isolate the effect of churning within the industry. The regressions use both rotation rate and large rotation rate to measure churning.

$Rot_{i,t}^{upstream}$ and $Rot_{i,t}^{downstream}$ are kept separately in the regressions. Because in most cases if industry j is industry i's supplier industry (in i's upstream), then j is also i's customer industry (in i's downstream). A change in industry j's churning would show up in both $Rot_{i,t}^{upstream}$ and $Rot_{i,t}^{downstream}$. Including them in the same regression would make regression result hard to interpret.

Table 1 reports the results of the above regressions. Column (1) and (2) report the regression results when $Rot_{i,t}^{upstream}$, the measures of churning for industries in the upstream of industry i, is an independent variable. β_1 and β_2 are estimated to be negative and significant for both measures of churning, indicating that an increase in churning within an industry or in its upstream industries associates with a decline in the industry's employment growth.

Column (3) and (4) report the regression results when $Rot_{i,t}^{downstream}$, the measures of churning for industries in the downstream of industry i, is an independent variable. Both coefficients are estimated to be negative and significant for both measures of churning, which means that an increase in churning within an industry or in its downstream industries is associated with a decline in the industry's employment growth.

It is worth noting that the coefficients of churning have a larger estimate when churning is measured by large rotation rate, which suggests that employment growth is more elastic to big turbulence in the profitability distribution.

1.3 Predictive analysis

The previous subsection shows that a higher churning of an industry and its downstream/upstream industries is associated with a slower employment growth. While churning might induce economic downturn in the short run, does it improve economic activities in the longer term?

To explore the relationship between churning and future employment growth, this subsection considers the following predictive regressions

$$\frac{1}{L}\sum_{l=1}^{L} eg_{i,t+l} = X_i + \gamma_t + \beta_1 \cdot Rot_{i,t} + \beta_2 \cdot Rot_{i,t}^{upstream} + \varepsilon_{i,t} \quad (1)$$

and

$$\frac{1}{L}\sum_{l=1}^{L} eg_{i,t+l} = X_i + \gamma_t + \beta_1 \cdot Rot_{i,t} + \beta_2 \cdot Rot_{i,t}^{downstream} + \varepsilon_{i,t} \quad (1)$$

$\frac{1}{L}\sum_{l=1}^{L} eg_{i,t+l}$ is the average employment growth between year $t+1$ to $t+L$. I let L be 5 and 10 (years), which correspond to the median run and the long run respectively.

Table 2 reports the result when $L = 5$, that is the dependent variable is the average employment growth in the next five years. As shown in columns (1) and (2), the coefficient of $Rot_{i,t}^{upstream}$ is estimated to be negative and significant, meaning that churning in an industry's upstream industries is associated with a slower employment growth in the next 5 years. Churning within the industry and in the downstream industries do not seem to have a significant impact on the median-term employment growth.

Table 3 reports the result when the dependent variable is the average employment growth in the next ten years. As shown in columns (3) and (4), the coefficient of $Rot_{i,t}^{downstream}$ is estimated to be positive and significant, meaning that churning in an industry's downstream industries is associated with a faster employment growth in the long term. Churning within an industry is associated with a faster long run growth when churning is measured by large

Table 2: Churning and median-term employment growth, L=5

	Upstream		Downstream	
Measures of churning	Rotation rate	large rotation rate	Rotation rate	large rotation rate
	(1)	(2)	(3)	(4)
$Rot_{i,t}$	0.00 (0.02)	0.01 (0.03)	-0.01 (0.01)	-0.02 (0.03)
$Rot_{i,t}^{upstream}$	-0.08^{**} (0.03)	-0.12^{***} (0.04)		
$Rot_{i,t}^{downstream}$			-0.03 (0.03)	-0.00 (0.04)
$\#\ of\ observations_{year \times industry}$	11×42	11×42	11×42	11×42

$\frac{1}{L}\sum_{l=1}^{L} eg_{i,t+l}$ =average industry employment growth, NIPA, 1999-2014
X_i = industry fixed effect, γ_t = year effect
$Rot_{i,t}$ = industry rotation rate in year t, Compustat
$Rot_{i,t}^{upstream}$ =rotation rate in the upstream industries in year t, Compustat
$Rot_{i,t}^{downstream}$ =rotation rate in the upstream industries in year t, Compustat
The analysis includes private non-farm industries that are in both Compustat and NIPA and have more than 8 firms in Compustat from 1999 to 2014
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.

rotation rate. Churning in the upstream industries do not seem to have a significant impact on the long-term employment growth.

To summarize the results in Tables 2 and 3, an increase in an industry's downstream industries' churning is associated with a faster employment growth of the industry in the long run; an increase in an industry's upstream industries' churning is associated with a slower employment growth of the industry in the median run. Moreover, the very sizable churning (as measured by the large rotation rate) within an industry predicts a faster industry employment growth in the long run. Although the correlation between churning and the median- and long-term labor market seems intriguing, this paper mainly focuses on the effect of churning on the labor market in the short run, while the median- and long-term effect of churning is above the scope of this paper.

1.4 Aggregate churning and unemployment rate

A recent literature on uncertainty shock finds that an increase in the volatility of idiosyncratic productivity, which can contribute to a higher churning in many settings, is usually

Table 3: Churning and long-term employment growth, L=10

	Upstream		Downstream	
Measures of churning	Rotation rate	large rotation rate	Rotation rate	large rotation rate
	(1)	(2)	(3)	(4)
$Rot_{i,t}$	0.03 (0.04)	0.06*** (0.02)	0.02 (0.03)	0.05*** (0.02)
$Rot_{i,t}^{upstream}$	0.01 (0.07)	−0.00 (0.03)		
$Rot_{i,t}^{downstream}$			0.16** (0.06)	0.09** (0.04)
$\#\, of\ observations_{year \times industry}$	6×42	6×42	6×42	6×42

$\frac{1}{L}\sum_{l=1}^{L} eg_{i,t+l}$ =average industry employment growth, NIPA, 1999-2014
X_i = industry fixed effect,γ_t = year effect
$Rot_{i,t}$ = industry rotation rate in year t, Compustat
$Rot_{i,t}^{upstream}$ =rotation rate in the upstream industries in year t, Compustat
$Rot_{i,t}^{downstream}$ =rotation rate in the upstream industries in year t, Compustat
The analysis includes private non-farm industries that are in both Compustat and NIPA and have more than 8 firms in Compustat from 1999 to 2014
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.

accompanied by a depression in economic activities. This subsection verifies that a higher churning of the aggregate economy is associated with a decline in the economic activities at the aggregate level.

To measure churning at the aggregate level, I construct an index of the aggregate rotation rate by aggregating the industry rotation rates weighted by industry value-added. Specifically, aggregate rotation rate Rot_t is defined as

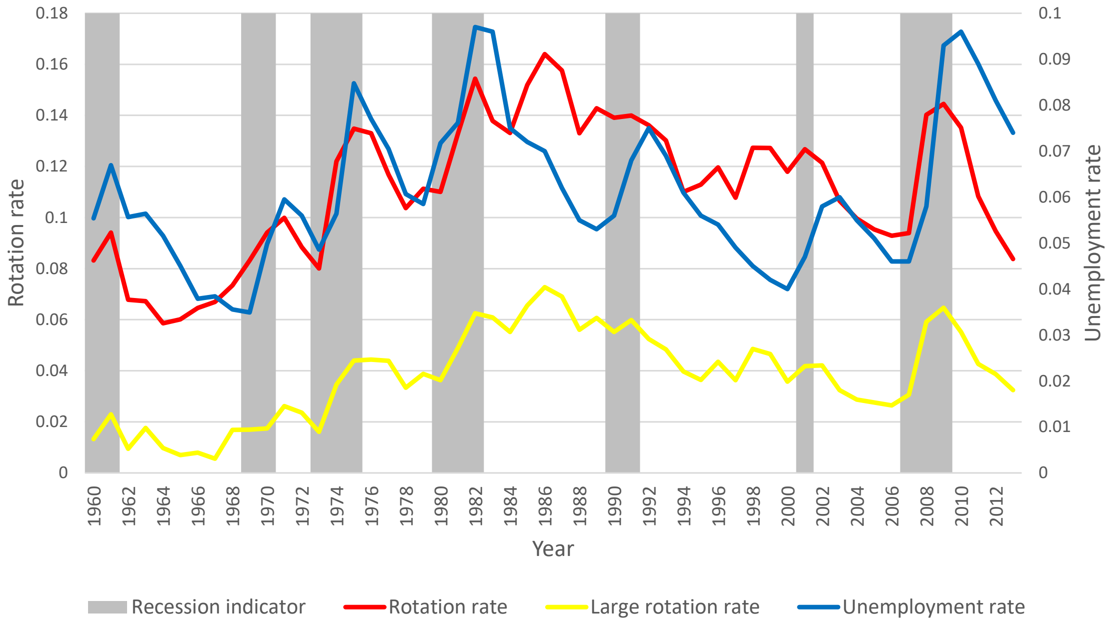
$$Rot_t = \sum_i Rot_{i,t} \cdot \frac{Value\ added_{i,t}}{GDP_t}$$

A higher aggregate rotation rate indicates a higher churning in the economy. Figure 1 plots the aggregate rotation rate and large rotation rate with the civilian unemployment rate.

Then I use an uni-variate regression to quantify the correlation between civilian unemployment rate and aggregate rotation rate:

$$Unemployment_t = \alpha + \beta \cdot Rot_t + \varepsilon_t$$

Figure 1: Measures of aggregate churning are correlated with the unemployment rate



The blue curve shows the civilian unemployment rate from the BLS; the red curve displays the rotation rate constructed from Compustat; the yellow curve displays the large rotation rate; Shaded areas correspond to NBER recessions.

Table 4 reports the results. In the regression, both rotation rate and large rotation rate are used to measure churning. I extract the cyclical components of churning and unemployment rate with HP filter. As shown in Table 4, coefficient β is significant and positive for both measures of churning, which indicates that an increase in churning of the aggregate economy is associated with a higher national unemployment rate.

To summarize the empirical findings of section 1, measures of churning are significantly associated with the condition of the labor market at both the industry level and the aggregate level. Particularly, measures of churning of an industry are associated with the condition of labor markets of its linked industries. Given these findings, it is reasonable to conjecture that churning contributes to the labor market fluctuations, and inter-firm linkage across industries serve as a propagation mechanism.

2 A simple model

Motivated by the empirical findings in section 1, this section demonstrates the main idea of this paper through a simple DMP model modified with firm inter-connectivity.

Table 4: Measures of aggregate churning are positively correlated with unemployment rate

Measure of churning	Coefficient	$Adj\ R-squared$	# of years
Rotation rate	0.42^{***} (0.08)	0.31	55
Large rotation rate	0.74^{***} (0.15)	0.31	55

$Unemployment_t$ = civilian unemployment rate t, BLS, 1960-2014
Rot_t = measure of churning in year t, Compustat
SE is in the parentheses.
1, 2, 3 asterisks denote significance at the ten, five, one percent level.

2.1 Baseline environment

Time is discrete. The economy has two distinct sets of firms indexed by A and B, so two labor markets. New production opportunities, corresponding to job vacancies (v_i) are created at cost χ. Each set of firms is associated with a labor market that is populated by a measure one of risk-neutral individuals who can be either employed in set i or unemployed and searching for a job. For simplicity, I assume workers cannot move across labor markets.[3]

At the beginning of each period, unemployed workers and job vacancies are matched in the frictional labor markets. Matching probability depends on the ratio of the number of vacancies to the number of unemployed workers. If unmatched, the firms exit the model and the workers remain unemployed. If matched, the new firms draw a type from H (high productivity) or L (low productivity) randomly with probability 50 percent and 50 percent. Then the firm instantaneously matches with a partner of the same type from the other set. I will show that the positive assortative matching (PAM hereafter) is the unique Nash equilibrium. For illustration, the simple model assumes that the inter-firm matching process is frictionless and a matched firm cannot separate from its partner.[4]

Production takes place after the matching processes. A firm's productivity depends on a firm's own type and its partner's type. I denote a firm's productivity as z_i^{jk} if it is in set i with type j cooperating with a k type partner. Firm and worker split output with fixed share τ and $1-\tau$. For simplicity I assume there is no unemployment insurance or disutility of working, so workers always prefer working to unemployment.

[3] As will be shown, in this model, the two sets always comove exactly and workers are indifferent between the two labor markets. The quantitative model in section (4) would describe the situation when the two sets do not comove and labor can move across labor markets.

[4] The quantitative model in section 4 relaxes the assumptions.

The key setting of the model is, after the production of goods, the type of each firm from set A switches with a Markov switching process, which is described by a Markov switching matrix Π_t with each row adds up to one. Here Π_t has a time subscript, meaning rotation rates are time-varying: in period t a type j firm switches to type k with probability ρ_t^{jk}. The rotation processes are i.i.d. across firms.

$$\Pi_t = \begin{bmatrix} \rho_t^{HH} & \rho_t^{HL} \\ \rho_t^{LH} & \rho_t^{LL} \end{bmatrix}$$

As the empirical counterpart of H type and L type is above the median and below the median in the productivity distribution, the measure of H type firms should always equal to the measure of L type firms, therefore Π_t is restricted to be symmetric with $\rho_t^{LH} = \rho_t^{HL}$.

My analysis focuses on shocks to the rotation rates. For simplicity, I ignore the rotation of type for firms in set B; that is, their types are permanent. However, their productivity can change due to the switching of their partner's type.

Following the canonical DMP model, worker-firm matches are destroyed exogenously at the end of each period with fixed rate δ. Upon destruction of the match, workers become unemployed in the same set since I assume that they cannot move across labor markets.

In sum, events unfold as follows: At the beginning of the period, the aggregate productivity and rotation rate of set A are observed. Firms post vacancy to match with unemployed workers. If successfully matched, firms randomly draw their type, either H or L, then match with a partner with the same type. If not matched, the firm exits the model. The production takes place right after the matching processes. Then the firms in set A rotate their type randomly with type rotation rate. At the end of each period, firms are destroyed and disappear from the model exogenously with a fixed rate δ.

2.2 Tightness ratios

In this subsection, I introduce some key notations of the labor market. Matching function $m(\mu_{i,t}, v_{i,t})$ determines how many matches are formed given the number of vacancies $v_{i,t}$ and the number of unemployed workers $u_{i,t}$. Labor market tightness ratio $\theta_{i,t}$ is the ratio of the number of vacancies to the number of unemployed workers in set i:

$$\theta_{i,t} = \frac{v_{i,t}}{u_{i,t}}, \quad i \in \{A,\ B\}$$

Job finding rate $\mu_{i,t}$ is the probability for an unemployed worker to match with a vacancy. Vacancy filling rate $q_{i,t}$ is the probability for a vacancy matching with an unemployed worker. The matching function is assumed to be Cobb-Douglas, hence $\mu_{i,t}$ ($q_{i,t}$) is increasing (decreasing) function of the $\theta_{i,t}$:

$$\begin{aligned} \mu_{i,t} &= \frac{m(u_{i,t}, v_{i,t})}{u_{i,t}} = \mu(\theta_{i,t}) \quad i \in \{A,\, B\} \\ q_{i,t} &= \frac{m(u_{i,t}, v_{i,t})}{v_{i,t}} = q(\theta_{i,t}) \end{aligned}$$

Unemployment rates are determined by the joint force of job creation and job destruction:

$$u_{i,t+1} = u_{i,t} - \underbrace{\mu_i(\theta_{i,t}) \cdot u_{i,t}}_{Job\ creation} + \underbrace{\delta \cdot e_{i,t}}_{Job\ destruction} \qquad i \in \{A,\, B\} \tag{1}$$

$u_{i,t}$ ($e_{i,t}$) is measure of unemployment (employment) with $u_{i,t} = 1 - e_{i,t}$. A lower tightness ratio decreases job creation and increases unemployment. The simple model aims to find the conditions under which increases in rotation rate ρ_A^{HL} cause an increase in unemployments $u_{A,t}$ and $u_{B,t}$ by reducing tightness ratios $\theta_{A,t}$ and $\theta_{B,t}$.

2.3 Firm's value function

Firm's production function and household's utility function are both linear. Labor is the only production input and is fixed as 1, so a firm's output equals to labor productivity $z_{i,t}^{jk}$. A firm's value function is simply the present value of profit, which is a fixed share τ of output. In set A, for a firm with type j and working with a partner with type k, its value function is

$$\begin{aligned} J_{A,t}^{jk} &= \tau \cdot z_{A,t}^{jk} + \beta(1-\delta) E_t \left(\rho_t^{jH} J_{A,t+1}^{Hk} + \rho_t^{jL} J_{A,t+1}^{Lk} \right) \\ j,\, k &\in \{H,\, L\} \end{aligned} \tag{2}$$

Similar to the notation of idiosyncratic productivity, the superscript jk indicates that the firm is type j and it cooperates with another firm of type k; subscript indicates the set of firm and time period.

The value is composed of the contemporary profit, τ fraction of output $z_{A,t}^{jk}$, plus the expected discounted value from the next period on. In the next period, with probability ρ_t^{jH}

and complementary probabilityρ_t^{jL}, the firm becomes H type and L type and therefore has value of $J_{A,t+1}^{Hk}$ and $J_{A,t+1}^{Lk}$.

Value of firms in set B is described by a similar equation:

$$J_{B,t}^{jk} = \tau \cdot z_{B,t}^{jk} + \beta (1-\delta) E_t \left(\rho_t^{kH} J_{B,t+1}^{jH} + \rho_t^{kL} J_{B,t+1}^{jL} \right) \tag{3}$$

Before characterizing the equilibrium of the labor market, let me introduce some useful notions.

Definition 1. (Supermodularity)

1, A production function is supermodular if

$$z_i^{HH} - z_i^{LH} > z_i^{HL} - z_i^{LL} \quad i \in \{A, B\}$$

2, A value function is supermodular if

$$J_i^{HH} - J_i^{LH} > J_i^{HL} - J_i^{LL} \quad i \in \{A, B\}$$

A production function is supermodular when marginal production is increasing in partner's type. Similarly for the supermodular value function. Another way to interpret supermodularity is that PAM yields higher aggregate productivity or value than cross matching between different type of firms, since an equivalent definition of supermodularity is $z_i^{HH} + z_i^{LL} > z_i^{HL} + z_i^{LH}$ (same for the value function).

Lemma 1.

1. A value function is strictly strictly increasing in partner's type if the production function is strictly increasing in partner's type.

2. A value function is supermodular if and only if the production function is supermodular.

With Lemma 1, it is easy to prove the following result.

Proposition 1. *PAM is the unique Nash equilibrium if the production function is strictly increasing in partner's type.*

The intuition of Proposition 1 is that, in the frictionless matching problem, when every firm prefers to match with the H type, H type firms only match with H type firms while L type firms are only left to match with L type firms.

2.4 Free entry condition and the equilibrium of the labor market

In each set, there are a large number of firms that can potentially post vacancies as long as they pay the cost χ. The value of posting a vacancy in set i is

$$V_{i,t} = -\chi + f_{i,t}\left(\underbrace{\frac{J_{i,t}^{HH}}{2}}_{draw\ H} + \underbrace{\frac{J_{i,t}^{LL}}{2}}_{draw\ L}\right) + [1-f_{i,t}]\max_{i}\left[E_t\left(V_{A,t+1}\right), E_t\left(V_{B,t+1}\right), 0\right] \quad i \in \{A,B\} \tag{4}$$

where $f_{i,t} = f(\theta_{i,t})$ denotes a firm's vacancy filling rate. If successfully matched with a worker, the firm draws H or L type with 50 percent and 50 percent probability and matches with a partner with the same type. The maximization of the expectation term implies that firms who fail to match with a worker can choose to post a vacancy in either market or to be inactive in the following period.

The equilibrium level of tightness ratio $\theta_{i,t}$ is determined by the free entry conditions

$$V_{i,t} = 0, \quad i \in \{A, B\}$$

or equivalently

$$\chi = f(\theta_{i,t}) \cdot \left(\frac{J_{i,t}^{HH}}{2} + \frac{J_{i,t}^{LL}}{2}\right) \quad i \in \{A, B\} \tag{5}$$

Take vacancy filling rate to the *LHS* of free entry condition equation 5 and plug equations 2 and 3 into *RHS*, free entry condition of set A then becomes

$$\begin{aligned} \frac{\chi}{f(\theta_{A,t})} &= \frac{1}{2}\cdot\left[\tau \cdot z_{A,t}^{HH} + \beta(1-\delta)E_t\left(J_{A,t+1}^{HH}\right)\right] \\ &+ \frac{1}{2}\cdot\left[\tau \cdot z_{A,t}^{LL} + \beta(1-\delta)E_t\left(J_{A,t+1}^{LL}\right)\right] \\ &- \frac{1}{2}\cdot\beta(1-\delta)\rho_t^{HL}E_t\left(\underbrace{J_{A,t+1}^{HH} + J_{A,t+1}^{LL} - J_{A,t+1}^{HL} - J_{A,t+1}^{LH}}_{mismatch\ loss}\right) \end{aligned} \tag{6}$$

As equation 6 is stochastic, I will use temporary shock to rotation rate in period t as an illustration. Suppose in period t, there is a temporary increase in the rotation rate ρ_t^{HL}. We can see that the first two rows of the RHS of equation 6 are not affected as the shock is temporary and values should return to the steady state in period $t+1$. The shock to ρ_t^{HL} would affect the last row only. Specifically, when $\left(J_A^{HH}+J_A^{LL}-J_A^{HL}-J_A^{LH}\right)$ is positive (the value function is supermodular), a temporary increase in ρ_t^{HH} would increase the efficiency loss caused by mismatch, which then translates into a decline in expected matching value. As $f\left(\theta_{A,t}\right)$ is decreasing in $\theta_{A,t}$, it implies a lower equilibrium tightness ratio $\theta_{A,t}$.

In addition to the effect on $\theta_{A,t}$, a higher rotation rate increases the share of mismatched firms in the next period. When the production function is supermodular, mismatched firms, on average, are less productive. This composition effect decreases the averaged TFP in set A in period $t+1$.

Set B has the same result. Although firms in set B do not switch their own types, their chance of being mismatched with their partners in set A is increased by the rotation shock. With supermodularity, a temporary increase in ρ_t^{HH} would lower set B's equilibrium level of tightness ratio and averaged TFP.

Proposition 2 generalizes the temporary shock to a persistent shock case and presents the main result of this section.

Proposition 2. *If the production function is strictly increasing in partner's type, and the persistence of the rotation shock is bounded below $|\psi|<1-\rho^{HL}-\rho^{LH}$, an increase in the rotation rate of one set*

1. *decreases the equilibrium tightness ratio of both sets*
2. *decreases the average productivity of both sets in the next period*

if and only if the production function is supermodular.

3 Testable microeconomic implication

As illustrated by proposition 2, the key element of the model is the supermodular production function. Although the idea of supermodular production function, or efficiency of positive assortative matching, has long been hypothesized by many studies[5], there is no direct microeconomic evidence on it. In this section, I test the assumption of supermodularity using the model's implication on firms' endogenous choice of inter-firm cooperation contract.

[5]For example, Becker (1973); Kremer (1993); Rhodes-Kropf and Robinson (2008); Shimer and Smith (2000)

3.1 Intuition of the test

The challenge of the test for supermodularity stems from the fact that firms would always initiate match with the same type partner as long as the production function is monotone, that is, every firm prefer to work with the best even if it's inefficient of doing so. Hence one cannot identify supermodularity from firms' choice of partner.

To overcome this challenge, I consider a slight extension to the simple model. Specifically, I allow firms to choose the duration of the cooperation contract before matching with partners. The intuition is that, when a firm is uncertain about its future type or its partner's future type, and when mismatch is inefficient, the firm does not want to make the commitment by choosing a long duration contract. In a mismatch, the H type always wants to leave, and the L type always wants to stay. Supermodularity means that on average, keeping a mismatch is an inefficient thing to do, because the opportunity cost to the H type is higher than the benefit to the L type.

In the empirical analysis, I use vertical integration and sourcing to proxy long- and short-duration contracts. Intuitively, sourcing is an option to hedge against the mismatch risk. In contrast, while vertical integration helps firms to avoid transaction costs,[6] it prevents firms from dissolving mismatches and re-allocating to new cooperation relationships. In the model, I show that supermodularity predicts two cross-industry patterns: first, fewer firms choose vertical integration in industries with higher churning; second, fewer firms choose vertical integration when their partners are in industries with higher churning. Then I verify that both predictions are consistent with data.

3.2 Extend the simple model

In this subsection, I extend the simple model by letting firms choose the type of cooperation contract either vertical integration (VI) or sourcing (SC). If a firm chooses vertical integration, it is permanently matched with its partner. If a firm chooses sourcing, it can sever mismatched partnership with probability ν_i then match with a new partner. Parameter ν_i is a number between zero and one depending on maturity of sourcing contract industry in industry i.[7] This rematch option comes with a price: each period, firms with sourcing contracts pay a transaction cost $\eta \sim iid\ N\left(\bar{\eta}, \bar{\sigma}^2\right)$, where η is a firm fixed effect that the firm can

[6]Common transaction costs include bargaining cost, contracting cost and hold-up risk, which has been discussed extensively in Transaction Cost Economics (TCE) literature pioneered by Coase (1937); Williamson (1979, 1981)

[7]For example, if a contract's maturity is 2 years, ν is 1/8 in a quarterly model

observe before choosing the contract; $\bar{\eta}$ and $\bar{\sigma}$ are mean and dispersion of transaction cost within industry i.[8] I assume that the draw of transaction cost η and the choice of cooperation contract takes place after a firm matches with a worker and before it matches with a partner firm.

As in the simple model there are two industries i and j. Only industry j has rotation, which is governed by a Markov switching matrix $\begin{bmatrix} \rho_j^{HH} & \rho_j^{HL} \\ \rho_j^{LH} & \rho_j^{LL} \end{bmatrix}$, while industry i does not have rotation.

Appendix A provides a detailed description of the extended model.

Proposition 3 establishes a testable cross sectional prediction of the model.

Proposition 3. *The share of firms choosing vertical integration is negatively correlated with rotation rates ρ_i^{HL} and ρ_j^{HL} if and only if the production function is supermodular.*

In figure 2, the left chart displays the correlation between vertical integration and the rotation rate in the linked industries, including both upstream and downstream industries. The right chart displays the correlation between vertical integration and the rotation rate in the same industry. Vertical integration is measured by ratio of industry value added to industry sales for 3-digit NAICS industries, which is a standard vertical integration index constructed with aggregate data series.[9] Both correlations are positive, which is consistent with the prediction of Proposition 3.

3.3 Vertical integration regression

The obvious concern of a cross-sectional test for Proposition 3 is that a variety of factors other than supermodularity - including economies of scale, access to credit, industry concentration, to name just a few - can determine the extent to which firms are willing to vertically integrate. It is hard to control for them all in the cross-sectional regression.

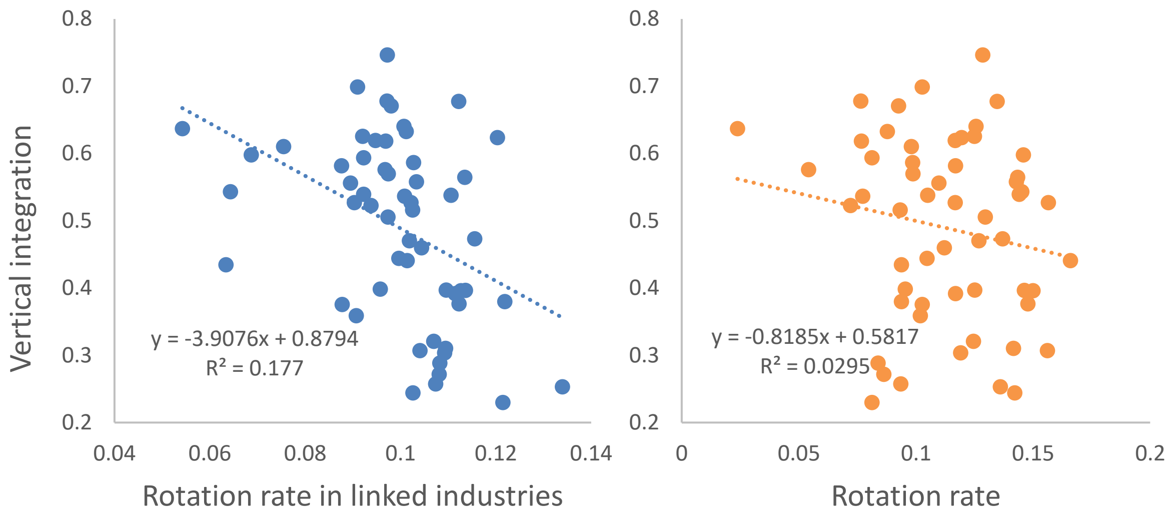
To address the above concern, I consider the following panel regression which includes industry fixed effect as an independent variable

$$VI_{i,t+1} = X_i + \gamma_t + \beta_1 Rot_{i,t} + \beta_2 Rot_{i,t}^{link} + \varepsilon_{i,t}$$

[8] As I do not have measurement of each industry's transaction costs or on their sourcing contracts' maturities, in the empirical analysis I treat ν, $\bar{\eta}$ and $\bar{\sigma}$ as constant across industries.

[9] See Acemoglu et al. (2010); Atalay et al. (2014) for examples of measuring vertical integration using firm level data.

Figure 2: Scatter chart of churning and vertical integration



The dependent variable $VI_{i,t}$ is the share of firms choosing vertical integration in industry i in year t. I proxy it with the ratio of industry value added to industry sales for 3-digit NAICS industries, which is a standard vertical integration index constructed with aggregate data.$VI_{i,t}$ counts both backward integration (purchase of supplier firms) and forward integration (purchase of customer firms).

X_i is the industry fixed effect that controls for the factors that potentially influences industry i's cross-industry vertical integration. I assume that the factors mentioned above should be relatively stable over time and are unlikely to change at the annual frequency. γ_t is the year effect that captures the change in $VI_{i,t+1}$ related to the aggregate business cycle.

$Rot_{i,t}$ is industry i's measure of churning in year t. $Rot_{i,t}^{link}$ is the weighted average of industry i's linked industries' (including industries in the downstream and in the upstream) churning. Weight of a linked industry j is the ratio of the gross flow of intermediate goods between industry i and industry j to industry i's gross trade of intermediate goods.

If production function is supermodular, β_1 and β_2 should be negative.

The regression results are reported in table 5. Column (1) shows the estimation result when churning is measured by rotation rate. The coefficient of the industry's churning is negative and significant at 5% significance, which indicates that a higher churning of an industry is associated with a slower vertical integration in the next year. The coefficient of the linked industries' rotation rate is also estimated to be negative, while the result is insignificant due to a large standard error.

Table 5: Measures of churning are negatively correlated with vertical integration

Measure of churning	Rotation rate	Large rotation rate
	(1)	(2)
$Rot_{i,t}$	-0.07^{**} (0.03)	-0.08 (0.05)
$Rot_{i,t}^{link}$	-0.01 (0.09)	-0.38^{***} (0.14)
$\underset{year \times industry}{\# of\ observations}$	16×48	16×48

SE is in the parentheses.

1, 2, 3 asterisks denote significance at the ten, five, one percent level.

X_i = industry fixed effect, γ_t = year effect

$VI_{i,t}$ = degree of vertical integration, $\frac{Value\ added_i}{Sales_i}$, *BEA IO tables 1997-2013*

$Rot_{i,t}$ = churning, *Compustat 1997-2013*

$Rot_{i,t}^{link}$ = churning of linked industries, *Compustat 1997-2013*

When churning is measured by large rotation rate as shown in column (2), both coefficients are estimated to be negative. In particular, the coefficient of linked industries' churning is significant at the 1% level.

In sum, the regression results show that an increase in the churning of an industry, and an increase in the churning of the linked industries, are associated with a lower degree of vertical integration of the industry in the next year. The results suggest that inter-firm matches are complementary, or the production function is supermodular.

4 A quantitative model

In this section, I embed the model studied in sections 2 and 3 into a general equilibrium, real business cycle (RBC) model. The primary purpose of this analysis is to evaluate the quantitative contribution of rotation shock to the business cycle, comparing with the standard shocks that have been intensively studied in the literature. We also want to ensure that the model can replicate the stylized cyclical behavior of macro aggregates, such as output, consumption, and investment.

4.1 Description of the quantitative model

I modify Andolfatto (1996) to include firm inter-connectivity and endogenous choice of inter-firm cooperation contract. With the model studied in sections 2 and 3, this model considers several additional features: There is capital, and both utility and production functions are concave; Firms match with partners in frictional inter-firm matching markets; Labor can move across labor markets.

The economy is populated by a continuum of households. There are two distinct labor markets or sets of firms: A and B. Production of final goods requires two intermediate inputs. Each set specializes in one intermediate goods. Firms from each set cooperate with partners from the other set: they produce final goods with intermediate goods then sell final goods in a competitive market. Representative household makes two decisions. First, it optimally allocates income to consumption and investment. Second, it sends its unemployed members to search for job vacancies in either labor market.

At the beginning of each period, firms post job vacancies to match with unemployed workers. The matching process is frictional and the search is random. If successfully matched, the firm becomes a *single firm* since it does not have a partner yet. A new single firm randomly draws a transaction cost $\eta \sim N\left(\bar{\eta}, \bar{\sigma}^2\right)$. Observing the transaction cost, the firm chooses its cooperation contract to be either vertical integration (VI) or sourcing (SC). Then the firm randomly draws a type—either H or L with probability 50 percent and 50 percent. I assume that single firms can produce final goods, but with low productivity.[10]

To have an inter-firm cooperation, single firms search for partners in frictional inter-firm matching market. As inter-firm search incurs no cost, every firm chooses to enter the inter-firm matching market. Search is directed, hence single firms are then divided into submarkets depending on their own types, their target partner's type, and the choice of cooperation contract. For example, the H type firms in set A who wants to vertically integrate with an L type partner would be located into the same submarket with the L type firms in set B who want to vertically integrate with an H type partner.[11] The matching technology is similar with the one in the labor market—the probability of finding partner depends on number of single firms from each set in each sub-market. If successfully matched, the firm becomes a *cooperative firm* since it has a partner to cooperate with.

[10]This can be micro-found in two ways. First is that single firm needs to produce both intermediate goods, one of which is not its comparative advantage. Second is that without inter-firm cooperation, single firm can only sell intermediate goods to a commodity market which is very competitive resulting in a lower profit.

[11]In theory, there can be eight submarkets: $(H-H)-VI$, $(H-L)-VI$, $(L-H)-VI$, $(L-L)-VI$, $(H-H)-SC$, $(H-L)-SC$, $(L-H)-SC$, $(L-L)-SC$. For example, the $(H-L)-VI$ sub-market accommodates the H type firms in set A who are looking for an L type partner and the L type firms in set B who are looking for an H type partner, both sides choose vertical integration as the cooperation contract. In the model, I show that under certain conditions, firms only search for same type partner; that is, there are only four submarkets operating in the Nash equilibrium: $(H-H)-VI$, $(L-L)-VI$, $(H-H)-SC$, and $(L-L)-SC$ markets.

Production takes place after the matching processes. In each set there are different categories of firms, depending on the firm's type and on whether the firm has a partner (in which case also on the partner's type and the contract they use). Production is organized by representative firms; they organize firms into production departments indexed by categories. They then rent capital from households and optimally allocate it to each production department.

After production, all firms rotate type according to a Markov switching process. Rotation rates are governed by the time varying and symmetric Markov switching matrices $\Pi_{A,t} = \begin{bmatrix} \rho_{A,t}^{HH} & \rho_{A,t}^{HL} \\ \rho_{A,t}^{LH} & \rho_{A,t}^{LL} \end{bmatrix}$ and $\Pi_{B,t} = \begin{bmatrix} \rho_{B,t}^{HH} & \rho_{B,t}^{HL} \\ \rho_{B,t}^{LH} & \rho_{B,t}^{LL} \end{bmatrix}$. For each firm-to-firm match, the two Markov switching processes are independent with each other. At the end of each period, firms are destroyed with fixed rate δ.

4.1.1 Household

The representative household's problem is:

$$\max E_t \left\{ \sum \beta^t \xi_{c,t} \left[log\left(C_t - \Psi_c \bar{C}_{t-1}\right) - \xi_{n,t} \sum_i \sum_j \sum_l n_{i,l,t}^j \right] \right\} \tag{7}$$

with $i \in \{A,B\}$, $j \in \{H,L,HH,HL,LH,LL\}$, $l \in \{VI,SC\}$.

Ψ_c is an external habit parameter, C_t denotes consumption, $\bar{C}_{t-1}$ is the economy's average consumption in period $t-1$. $n_{i,t}^j$ is the measure of employment in production department indexed by type j in set i, and has cooperation contract l. There are two preference shocks: shock to the discount rate $\xi_{c,t}$ and shock to the disutility of labor $\xi_{n,t}$.

The household's problem is subject to:

(1) The transition rules of unemployment and employment which are described in the appendix.

(2) The budget constraint

$$C_t + T_t + I_t = \int \sum_i \sum_j \sum_l w_{i,l,t}^j(\eta) \cdot n_{i,l,t}^j(\eta)\, d\eta + z \cdot \sum_i u_{i,t} + r_{k,t} K_t + D_t \tag{8}$$

The representative household's income is composed of wage income (the integration term), the unemployment insurance $z \cdot u_t$, capital rent $r_{k,t} K_t$, and dividend D_t. Household allocates income to consumption C_t, investment I_t, and receives a lump-sum tax T_t.

(3) The accumulation of capital

$$K_{t+1} = (1-d)K_t + \left[1 - S\left(\xi_{I,t}\left(\frac{I_t}{I_{t-1}}\right)\right)\right] I_t \tag{9}$$

Function S models the investment adjustment cost which equals zero in the steady state.

4.1.2 Representative firm

In each set, the representative firm operates production departments indexed by type j and cooperation contract l. Representative firms make four decisions: (1) rent capital from household then optimally allocate to each department, (2) post vacancy to match with worker,[12] (3) send single firms to search for partners in the inter-firm matching market. For the third decision, I conjecture then verify that in the Nash equilibrium, single firms only search for same type partners.

In set i, the representative firm maximizes the value function:

$$\begin{aligned} J_i\left(\left\{n^j_{A,l}(\eta)\right\}, \left\{n^j_{B,l}(\eta)\right\}, \Omega\right) &= \max_{v_i, \{k^j_i\}} x \sum_j z^j_i \left\{ \sum_l \int \left[\left(k^j_{i,l}(\eta)\right)^{\alpha} \left(n^j_{i,l}(\eta)\right)^{1-\alpha} \right] \right\} \\ &\quad \underbrace{- \sum_j \left\{ \int_{\Xi} \left[w^j_{i,SC}(\eta) \cdot n^j_{i,SC}(\eta) + r_k k^j_{i,SC}(\eta) + \eta \right] d\eta \right\}}_{Sourcing} \\ &\quad \underbrace{- \sum_j \left\{ \int_{\Xi^C} \left[w^j_{i,VI}(\eta) \cdot n^j_{i,VI}(\eta) + r_k k^j_{i,VI}(\eta) \right] d\eta \right\}}_{Vertical\ integration} \\ &\quad - v_i \cdot \chi + \beta E\left[\frac{\lambda'}{\lambda} J_i\left(\left\{n^j_{A,l}(\eta)\right\}, \left\{n^j_{B,l}(\eta)\right\}, \Omega'\right) \right] \end{aligned} \tag{10}$$

with $i \in \{A, B\}$, $j \in \{H, L, HH, HL, LH, LL\}$, $l \in \{VI, SC\}$, and subject to the matching

[12] Noticing that firm-worker match type is revealed only after the match is formed, firms post one type of vacancy only.

technology in the labor market

$$v_i = u_i \cdot \theta_i \tag{11}$$

The production function is Cobb-Douglas. TFP is determined by the firm's idiosyncratic productivity z_i^j and aggregate TFP x. Firms need to pay the cost on labor, capital, job vacancy. In addition, for firms who use SC as the inter-firm cooperation contract, they need to pay transaction cost η. I use Ξ to denote the set of firms who use SC. Naturally, Ξ^C contains the firms using VI, who do not pay the transaction cost.

4.1.3 Nash Bargaining and wage determination

Following the DMP search and matching model, wage is determined by Nash bargaining. In each period firms and workers set wages to solve the following Nash bargaining problem:

$$w_{i,l}^j(\eta) = arg\max_w \left(\frac{\frac{\partial V}{\partial n_{i,l}^H(\eta)} - \frac{\partial V}{\partial u_i}}{\lambda} \right)^{1-\tau} \left(\frac{\partial J_i}{\partial n_{i,l}^H(\eta)} \right)^{\tau} \tag{12}$$

In the above equation, time subscripts are omitted. τ is the firm's bargaining share. $\frac{\partial V}{\partial n_{i,l}^H(\eta)}$ and $\frac{\partial V}{\partial u_i}$ are the representative household's marginal value of employment and unemployment respectively. $\frac{\partial J_i}{\partial n_{i,l}^H(\eta)}$ is firm's marginal value of employment.

4.1.4 Economy resource constraint and shocks

I close the model by presenting the economic resource constraint and the processes of shocks.

The economy can allocate output—the aggregate of all departments' output net of transaction cost—to consumption, investment and vacancies. Thus the economy resource constraint is:

$$C + I + \chi \sum_i v_i = x \sum_i \sum_j \sum_l z_i^j \left(k_{i,l}^j(\eta) \right)^{\alpha} \left(n_{i,l}^j(\eta) \right)^{1-\alpha} - \sum_i \sum_j \left(\int_{\Xi} \eta d\eta \right) \tag{13}$$

I include exogenous shocks to six variables in the model. Besides the shocks to the rotation rates ρ_A^{HH} and ρ_B^{HH}, the model has the shocks to aggregate TFP, to investment adjustment cost ξ_I, to inter-temporal preference ξ_C and to labor disutility ξ_L. The shocks are governed by $AR(1)$ processes that are independent to each other:

$$
\begin{aligned}
log\left(\xi_{w,t}\right) &= \rho log\left(\xi_{w,t-1}\right)+\sigma_w\varepsilon_{w,t}, \quad \varepsilon_{w,t}\sim N\left(0,1\right) \\
w &= x,I,C,L
\end{aligned}
$$

The values of all shock variables are observable to economic agents at beginning of each period.

4.2 The sufficient condition for PAM

In the models studied in sections 2 and 3, without inter-firm search friction, PAM is always the Nash equilibrium as long as the production function is increasing in partner's type. With inter-firm search friction, however, monotonicity is not sufficient for PAM: there exists a possibility that a large amount of of L type single firms are willing to chase for just a few H type single firms, while the few H type firm might be happy to match with those L type if the matching probability is high enough. In proposition 4, I show the sufficient condition for PAM to be the Nash equilibrium in the friction environment.

Proposition 4. *PAM is Nash equilibrium if*

$$
\left(\frac{J_{A,l}^{HH}-J_{A,l}^{H}}{J_{A,l}^{HL}-J_{A,l}^{H}}\right)^{\frac{1}{\alpha_2}}\times\left(\frac{J_{B,l}^{LH}-J_{B,l}^{L}}{J_{B,l}^{LL}-J_{B,l}^{L}}\right)^{\frac{1}{1-\alpha_2}}>1 \quad l\in\{VI,\ SC\} \tag{14}
$$

$$
\left(\frac{J_{B,l}^{HH}-J_{B,l}^{H}}{J_{B,l}^{HL}-J_{B,l}^{H}}\right)^{\frac{1}{1-\alpha_2}}\times\left(\frac{J_{A,l}^{LH}-J_{A,l}^{L}}{J_{A,l}^{LL}-J_{A,l}^{L}}\right)^{\frac{1}{\alpha_2}}>1 \tag{15}
$$

where $J_{i,l}^{k}=\frac{\partial J_i}{\partial n_{i,l}^{k}}$

When solving and estimating the model, I first impose PAM to be the Nash equilibrium, then verify that the inequality conditions in proposition 4 hold.

4.3 Estimation of the quantitative model

In this subsection, I estimate the model described above and quantitatively assess the effect of rotation shock.

4.3.1 Data and methodology

The sample period is 1969Q1 to 2013Q4. The estimation uses eight variables: rotation rates of goods-producing sector (set A) and service-providing sector (set B), real interest rate, aggregate unemployment rate, aggregate job opening rate, growth rates of real investment and real consumption per capita, growth rate of per hour real wage. Data sources are described in the Appendix. Keeping the same notation as in the description of the model above, and writing Δ to indicate first differences, the full vector of observables is

$$\left[\rho_{A,t}^{HL}, \rho_{B,t}^{HL}, R_t, u_t, v_t, \Delta log\left(I_t\right), \Delta log\left(C_t\right), \Delta log\left(w_t\right)\right]$$

I first solve and re-scale the model then log-linearize the non-linear model around a deterministic steady state and write the linearized equilibrium conditions in a state-space form. The resulting linear rational expectations model can then be solved by methods such as in Sims (2002).

The use of eight series of observables requires the inclusion of at least eight independent sources of variation. I consider six independent shocks: aggregate TFP shock, two rotation shocks, marginal efficiency of investment (MEI) shock, discount rate shock, and labor disutility shock. As the number of observable is larger than the number of shocks, the model cannot be identified. So I include observation errors for two observable, $\rho_{A,t}^{HL}, \rho_{B,t}^{HL}$, as additional disturbances. In the estimation, I made the restriction that the total surplus is positive for any firm-worker match, so that no firm-worker match wants a separation.

In the implementation of the Bayesian estimation procedure, I use the Kalman-filter to evaluate the likelihood function of the observable. I then combine the likelihood function with the prior distribution of the model's parameters to obtain the posterior distribution. I then evaluate the posterior distribution using the random-walk Metropolis-Hasting algorithm. Further details on the computational procedure can be found in An and Schorfheide (2007).

4.3.2 Calibration

Firstly, I calibrate some parameters based on the typical values used in calibration studies. Inter-firm matching is assumed to be symmetric: the inter-firm matching elasticity α_0 is set to 0.5. I normalize the maximum TFP z_i^{HH} as 1. As the empirical rotation rate is low in the steady state (5% annually), the majority of firms fall into the HH and LL category. Hence

Table 6: Calibration

Description	Parameter	Calibration	Target
Firm-firm match elasticity	α_2	0.5	Inter-sector Symm.
Max TFP	z^{HH}	1	Normalization
Ratio of productivities	z^{LL}/z^{HH}	0.4	Syverson (2004)
Vacancy cost	χ	1	Normalization
Unemployment insurance	z	0.25	Department of Labor
Exog separation rate	δ_1	0.05	Shimer (2012)
Discount rate	β	0.99	2% real interest rate
Capital share	α	0.34	Capital income share in the US
Capital depreciation rate	δ	0.025	10% annual depreciation rate

I set z_i^{LL} to 0.4, which targets to inter-quartile ratio of productivity for manufacturing firms in U.S. (Syverson (2011)).[13] Vacancy posting cost χ is normalized as 1. Following Shimer (2012), I set the exogenous separation rate δ_1 as 0.05. Ds in standard models, discount rate β is set to 0.99, capital share α and depreciation rate of capital δ are set to 0.34 and 0.025. The calibrations are summarized in Table 4.3.2.

The other parameters are estimated using Bayesian method. I discuss the selection of prior in the appendix.

4.3.3 Parameter estimates

Tables 7 and 8 compare the posterior distributions to the priors. The posterior estimates of the parameters that are present in common DSGE studies are in line with previous studies and I skip the discussion on them.

Productivity parameters

Productivities of single firms in set A z_A^H and z_A^L are estimated to be 0.46 of z^{HH} and 0.50 of z^{LL}. This means that firms can double their productivity when matching with a same type partner. For mismatched cooperative firms in set A, their productivities z_A^{HL} and z_A^{LH} are 0.43. These results imply that an H type firm would find it more productive to work alone than to

[13] It also implies that the inter-quartile ratio of the wage distribution is 0.62 at the posterior mode.

Table 7: Estimation result I

Parameter	Description	Prior			Posterior	
		Type	Mean	Std	Mode	90% interval
γ_A^H	z_A^H/z_A^{HH}, z_A^L/z_A^{LL}	beta	0.5	0.1	0.46	[0.37,0.53]
γ_B^H	z_B^H/z_B^{HH}, z_B^L/z_B^{LL}	beta	0.5	0.1	0.50	[0.44,0.61]
γ_A^{HL}, γ_A^{LH}	z_A^{HL}/z_A^{HH}, z_A^{LH}/z_A^{HH}	beta	0.6	0.15	0.42	[0.47,0.78]
γ_B^{HL}, γ_B^{LH}	z_B^{HL}/z_B^{HH}, z_B^{LH}/z_B^{HH}	beta	0.6	0.15	0.43	[0.13,0.59]
ν	Separation probability for SC	beta	0.25	0.05	0.13	[0.12,0.15]
$\bar{\eta}$	Mean of transaction cost	normal	0	0.2	-0.13	[-0.14,-0.13]
$\bar{\sigma}$	Std. of transaction cost	inv gamma	2.0	0.2	1.01	[1.00,1.01]
ϕ_A	A's matching efficiency	beta	0.7	0.1	0.73	[0.67,0.78]
ϕ_B	B's matching efficiency	beta	0.7	0.1	0.75	[0.71,0.81]
ψ	Inter-firm ME	beta	0.7	0.25	0.76	[0.65,0.96]
τ	Bargaining share of firm	beta	0.5	0.1	0.92	[0.87,0.94]
$\frac{\xi_L}{\lambda}$	Mean of labor disutility	beta	0.2	0.1	0.11	[0.08,0.12]
ψ_c	Habit persistence	beta	0.5	0.1	0.04	[0.04,0.05]
S_2	Investment adj. cost	normal	4	2	4.17	[4.10,4.18]
$100\times\mu^*$	Trend TFP growth	normal	0.4	0.1	0.14	[0.14,0.15]

cooperate with an L type partner; an L type firm would barely gain any productivity by working with an H type partner, in contrast with working with an L type partner. Productivities of firms in set B have very similar estimates.

With the above results, we can verify that the conditions in propositions 4 are satisfied at the posterior mode, so that in the Nash equilibrium, single firms initiate match with same type partners. Specifically, I confirm that H type single firms prefers to match with H type partners, which guarantees that no firm would deviate from the Nash equilibrium in which firms only search for same type partners. Moreover, the marginal value function of firms is supermodular at the posterior mode, which implies that mismatch is estimated to be inefficient.

Parameters of the labor market and the cooperation contract

Labor market matching efficiencies ϕ_A and ϕ_B have posterior mode of 0.73 and 0.75, which are close to the prior mean and imply a job finding rate of 0.56. The prior mean of the two

Table 8: Estimation result II

Para	Description	Prior			Posterior	
		Type	Mean	Std	Mode	90% interval
$10 \times \bar{rot}_A$	Mean of *Rot* in *A*	beta	1	0.1	0.12	[1.12,1.42]
$10 \times \bar{rot}_B$	Mean of *Rot* in *B*	beta	1	0.1	0.13	[1.29,1.52]
σ_{rot_A}	Std of shock to *Rot* in *A*	inv gamma	0.005	0.2	0.40	[0.37,0.43]
σ_{rot_B}	Std of shock to *Rot* in *B*	inv gamma	0.005	0.2	0.38	[0.35,0.40]
σ_z	Std of aggregate TFP	inv gamma	0.005	0.2	0.71	[0.68,0.75]
σ_{ξ_c}	Std of discount rate shock	inv gamma	0.005	0.2	1.97	[1.81,2.14]
σ_{ξ_I}	Std of investment shock	inv gamma	0.005	0.2	0.88	[0.78,0.98]
σ_{ξ_l}	Std of disutility shock	inv gamma	0.005	0.2	39.12	[31.28,46.53]
ρ_{Rot_A}	Pers of shock to *Rot* in *A*	beta	0.5	0.1	0.92	[0.90,0.94]
ρ_{Rot_B}	Pers of shock to *Rot* in *B*	beta	0.5	0.1	0.91	[0.89,0.92]
ρ_z	Pers of aggregate TFP	beta	0.5	0.1	0.95	[0.94,0.97]
ρ_{ξ_c}	Pers of discount rate shock	beta	0.5	0.1	0.75	[0.72,0.76]
ρ_{ξ_I}	Pers of investment shock	beta	0.5	0.1	0.37	[0.41,0.45]
ρ_{ξ_l}	Pers of disutility shock	beta	0.5	0.1	0.94	[0.94,0.95]

parameters is set to match the steady state unemployment rate; the estimates of labor market matching efficiency well capture the average unemployment rate in the data.

At its posterior mode, firm's bargaining share is 0.92. This is similar with the result of Lubik (2009), who estimated firm's bargaining share to be 0.97. As I impose the Hosios condition, the labor market matching elasticity is also 0.92. Labor disutility has a posterior mode of 0.11. Combining unemployment insurance with the adjusted labor disutility, the implied flow value of unemployment is about 37% of the average wage in the steady state.

It is well known that in canonical DMP model driven by exogenous TFP shocks only, an extremely high flow value of unemployment and low firm's bargaining share are often needed to generate realistic volatility of unemployment.[14] In this model, rotation shocks and labor disutility shocks emerge as the main drivers of the labor market fluctuations. Both shocks can generate volatile unemployment without the help of a high flow value of unemployment and a low bargaining's share for firm, so the model estimated a high firm's bargaining share and a low flow value of unemployment.

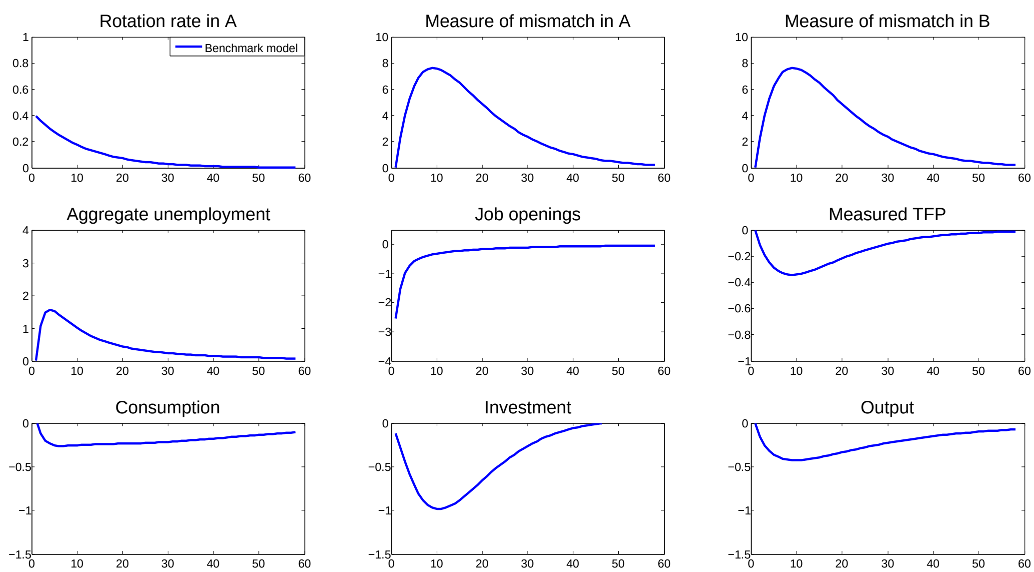
[14] Hornstein et al. (2005) provides an in-depth discussion.

The separation probability of sourcing contract has a posterior mode of 0.12, which is much smaller than the prior mean and indicates that the data suggests a large friction preventing mismatched firms from reallocating to more efficient partnerships. In the posterior mode, the transaction cost distribution has a mean of -0.13 and standard deviation of 1.01, which implies the ratio of industry value added to industry sales to be 0.45, which is close to 0.53 as measured in the BEA 2007 input-output table.

4.3.4 Impulse response analysis

Consider the reaction of the economy to a positive rotation shock to set A; that is, a persistent increase in the probability of switching type for firms in set A. The lines in Figure 4.3.4 are the impulse responses, in percentage deviations from the steady state, to a one standard deviation increase in rotation rate.

Figure 3: Impulse response to 1 std. of rotation shock in set A



The panels plot the percent deviation from the steady state of each induced by a one standard deviation shock to the rotation rate in set A.

The shock increases rotation rates ρ_A^{HL}. A higher rotation rate generates a labor market slack in which unemployment increases and vacancies drop. This is because higher rotation rate increases the probability of mismatch between different types of firms. Notice that marginal value function is supermodular at the posterior mode, hence the risk of mismatch negatively affects firms' expected values which reduces their incentive to create jobs.

A higher rotation rate also leads to a gradual decline in measured TFP by increasing the share of mismatched firms, who have lower production efficiency on average.

The increase in churning leads to a decline in macro aggregates such as consumption, investment and output through three channels. First, increase in churning hurts aggregate production efficiency by misallocating firms to inefficient partnerships, which causes a decline in aggregate output. Second, similar to the response of measured TFP, rotation shock leads to a decline in MPK, which makes households hold back investment. This then translates into a gradual decline in capital stock. Lastly, a weakening labor market reduces employment, which contributes to the decreasing output.

4.3.5 Comparing rotation shock with TFP shock

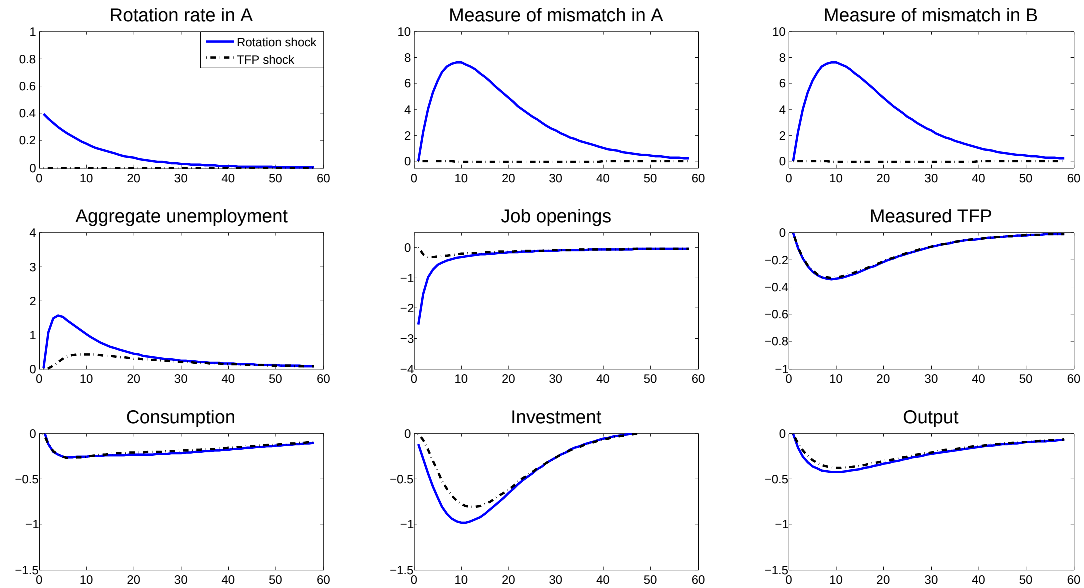
In the model, rotation shock gives rise to endogenous change in TFP. The natural question is: is there any quantitative difference between rotation shock and exogenous aggregate TFP shock in terms of their impact on the labor market variables and other macro aggregates? If so, can rotation shock speak to the Shimer puzzle (Shimer (2005))? As pointed out by Shimer (2005), canonical DMP model propagated by exogenous TFP shocks under-predicts the volatility of the labor market variables relative to TFP[15] and over-predicts the correlation between the labor market variables and TFP.

To answer the above questions, I simulate a model with exogenous aggregate TFP shock only, then compare with the impulse response to rotation shock in Figure 3. To make the two cases comparable, I set the path of TFP shock so that the measured TFP is identical in the two cases.

The responses of labor market variables are very different in the two cases. First, shock to rotation rate generates a much stronger effect in the labor market than TFP shock. Moreover, TFP shock generates perfect comovement between the labor market variables and the measured TFP, which is inconsistent with data. In contrast, rotation rate generates a sudden drop in job openings and tightness ratio and a gradual U-shape decline in the measured TFP, which breaks its perfect correlation with the unemployment rate and job openings.

[15]There is a vast literature on this, see Hall (2005); Hornstein et al. (2005); Mortensen and Nagypal (2007); Hall and Milgrom (2008); Hagedorn and Manovskii (2008); Fujita and Ramey (2009); Gertler and Trigari (2009) for example.

Figure 4: Compare rotation shock with TFP shock



The panels plot the percent deviation from the steady state induced by either one standard deviation shock to the rotation rate or a series of shocks to aggregate TFP.

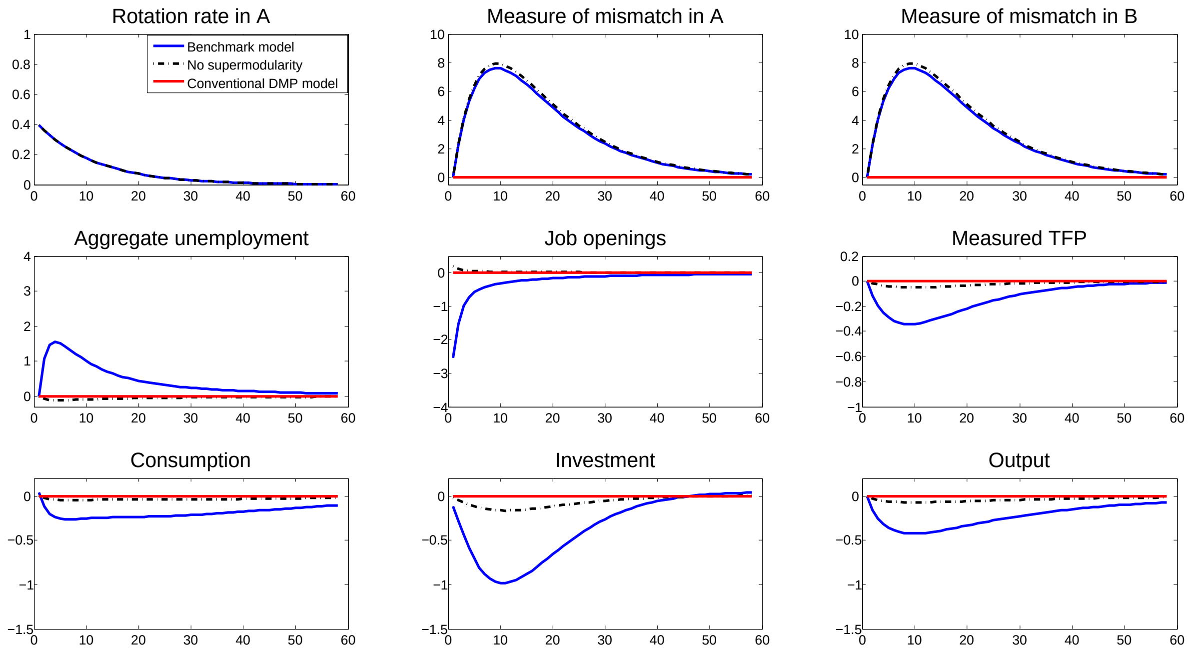
4.3.6 The role of firm inter-connectivity and supermodularity

To understand the role of firm inter-connectivity and supermodularity in the quantitative model, I simulate two counterfactual models, one canonical DMP model with no firm inter-connectivity, the other one with firm inter-connectivity yet without supermodularity. Then I compare the effect of rotation shock on these two models to the benchmark case. In Figure 5, red lines are impulse response functions in the model with no firm inter-connectivity; black dashed lines are for model without supermodularity; blue lines correspond to the benchmark case with both firm inter-connectivity and supermodularity.

When firm inter-connectivity is removed, my model becomes identical to the canonical DMP model and rotation shock does not have any aggregate effect. In this case, churning only shifts the dispersion between the marginal values of different types. However, the mean of marginal values, which determines the aggregate job creation, is not affected.

When there is firm inter-connectivity, but the marginal value function is not supermodular, positive rotation shock does not increase unemployment. In fact, according to the impulse response function, higher churning results in higher job creation and lower unemployment. This is a general equilibrium effect. The intuition is as follows: while the marginal

Figure 5: Impulse responses to rotation shock in counterfactual analysis



The panels plot the percent deviation from the steady state of each induced by a one standard deviation shock to the rotation rate in set A.

value function is not supermodular, firm's TFP is still supermodular; that is, we still have $z_i^{HH} - z_i^{HL} > z_i^{LH} - z_i^{LL}$. Therefore a higher churning leads to diminished output and a decline in consumption growth. As a result, discount factor $\beta \frac{u_C'}{u_C}$ rises and firms are more willing to allocate resource to the activities generating future profit, including the recruiting of worker. While this general equilibrium effect also appears in the benchmark model, it is dominated by the mismatch risk effect.

4.3.7 Variance decomposition

To evaluate the contribution of rotation shock to the business cycle, I conduct variance decomposition at business cycle frequency and report the results in Table 9. Each row reports the shares of fluctuations explained by the shocks at frequencies between 6 and 32 quarters, computed by a bandpass filter as in Stock and Watson (1999).

Rotation shocks account for a significant fraction of variation of all macro variables listed in the table. In particular, rotation shocks contribute to 27 percent of labor market fluctuations and 32 percent of fluctuation of aggregate output. There are two main reasons for the data to assign such a significant role to rotation shocks. First, as displayed in the impulse re-

Table 9: Variance decomposition at the business cycle frequency

Variance decomposition	TFP	DR	DL	MEI	Rot_A	Rot_B
Unemployment rate	0.03	0.02	0.67	0.01	0.14	0.13
Tightness ratio	0.02	0.02	0.68	0.01	0.14	0.13
Output	0.62	0.00	0.06	0.00	0.17	0.15
Measured TFP	0.48	0.00	0.01	0.00	0.27	0.24
Investment	0.05	0.16	0.04	0.45	0.25	0.22
Consumption	0.25	0.08	0.03	0.37	0.14	0.12

TFP is aggregate total factor productivity. DR is discount rate. DL is disutility of labor. MEI is marginal efficiency of investment. Rot_i is rotation rate for set i.

sponse functions, rotation shocks have a strong effect on both labor market variables as well as macro aggregates without relying on extreme values of parameters. The second reason is that rotation shocks correctly generate co-movement of all observables, hence it is preferred by the data.

Shocks to TFP have a minor effect on unemployment rate and tightness ratio due to the lack of amplification of productivity shocks in the model, while accounting for large fractions of the variances of macro aggregates such as output and measured TFP. Shocks to the labor disutility have a major effect on labor market variables. A positive shock to the labor disutility increases the option value of unemployment, which makes the total surplus of potential worker-firm matches shrink and hence reduces firm's incentive to create jobs. Shocks to the discount rate, or preference shocks, contribute to a large fraction of investment and consumption fluctuations. It also explains a moderate fraction of labor market fluctuations. Shocks to the marginal efficiency of investment have a large effect on consumption and investment, yet has little contribution to the other variables' fluctuations.

5 Conclusions

This paper is motivated by two empirical facts. The first is that during downturns of economic activity, there is an increase in churning of firms' rankings in the profit distribution. Second, a higher churning of an industry is usually accompanied by an economic downturn within the industry and in its linked industries. Based on the two facts, this paper studies the effect of churning on the business cycle emphasizing the role of firm inter-connectivity.

Specifically, I develop and estimate a Diamond-Mortenson-Pissarides search and matching model featured by inter-firm matching. When firms have the comparative advantage of working with same type partner, an increase in the churning of an industry leads to a recession within the industry and in its linked industries. Quantitative analysis indicates that the variations in churning is one of the major sources of persistent and joint movements in unemployment and other macro variables.

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Appendix

A Extended model with endogenous choice of contract

A.1 Value function

In industry i, value functions for firms choosing sourcing as cooperation contract are:

$$\begin{aligned} J_{i,SC}^{HH}(\eta) &= \tau\cdot\left(z_i^{HH}-\eta\right) \\ &+ \beta(1-\delta)E_t\left[\rho_j^{HH}J_{i,SC}^{HH}(\eta)+(1-\nu)\rho_j^{HL}J_{i,SC}^{HL}(\eta)+\nu\rho_j^{HL}J_{i,SC}^{HH}(\eta)\right] \end{aligned} \tag{A.1}$$

$$\begin{aligned} J_{i,SC}^{LL}(\eta) &= \tau\cdot\left(z_i^{LL}-\eta\right) \\ &+ \beta(1-\delta)E_t\left[\rho_j^{LL}J_{i,SC}^{LL}(\eta)+(1-\nu)\rho_j^{LH}J_{i,SC}^{LH}(\eta)+\nu\rho_j^{LH}J_{i,SC}^{LL}(\eta)\right] \end{aligned} \tag{A.2}$$

The time subscripts are omitted in the above equations. The value of H type firms is composed of contemporary profit and continuation value in the next period; with rotation rate ρ_j^{HL}, the firm's partner becomes L type, and with probability ν, the firm rematches with another H type partner. The subscript SC denotes that the firm uses sourcing as cooperation contract.

The value functions for firms with vertical integration are same as those in the original simple model. A firm that chooses vertical integration is exempt from transaction cost η, but does not have a rematch option when mismatch occurs.

$$J_{i,VI}^{HH} = \tau\cdot z_{i,t}^{HH}+\beta(1-\delta)E_t\left(\rho_j^{HH}J_{i,VI}^{HH}+\rho_j^{HL}J_{i,VI}^{HL}\right) \tag{A.3}$$

$$J_{i,VI}^{LL} = \tau\cdot z_{i,t}^{LL}+\beta(1-\delta)E_t\left(\rho_j^{LL}J_{i,VI}^{LL}+\rho_j^{LH}J_{i,VI}^{LH}\right) \tag{A.4}$$

A.2 The choice of contract

The focus of this section is firms' choice of contract and how it relates to churning and its associated mismatch risk. I first show that firms' choice of contract can be categorized by a threshold η_i^*: firms with transaction cost η beyond η_i^* would choose vertical integration; ones with η below η_i^* would choose sourcing.

I conjecture then verify that in industry i, there exists a threshold η_j^* that makes a firm indifferent between sourcing and vertical integration. η_j^* should satisfy

$$\frac{1}{2}\left(J_{i,VI}^{HH}+J_{i,VI}^{LL}\right)=\frac{1}{2}\left[J_{i,SC}^{HH}\left(\eta_j^*\right)+J_{i,SC}^{LL}\left(\eta_j^*\right)\right]$$

In the appendix, I show that η_i^* is determined by

$$\tau\cdot\eta_i^*=\beta\left(1-\delta\right)g\left(\rho_j^{HL},\nu\right)\cdot\left(z_i^{HH}+z_i^{LL}-z_i^{HL}-z_i^{LH}\right) \tag{A.5}$$

with $\frac{\partial g\left(\rho_j^{HL},\nu\right)}{\partial\rho_j^{HL}}>0$ and $\frac{\partial g\left(\rho_j^{HL},\nu\right)}{\partial\nu}>0$

The LHS of equation A.5 is the cut-off firm's transaction cost that it can save with vertical integration. The RHS is the hedgable part of mismatch risk it can hedge with sourcing, which depends on rematch probability, industry j's rotation rate and its mismatch loss. Function g is defined in appendix; it is increasing in ρ_j^{HL} and decreasing in ν. The intuition of equation A.5 is that the cut-off firm should find transaction cost exactly equal to the hedgable part of mismatch risk.

I denote $\tilde{z}_i=z_i^{HH}+z_i^{LL}-z_i^{HL}-z_i^{LH}$, and rewrite equation A.5 as

$$\eta_i^*=\frac{\beta\left(1-\delta\right)g\left(\rho_j^{HL},\nu\right)\cdot\tilde{z}_i}{\tau}$$

Proposition 5 shows that a firm's choice of contract can be categorized by the threshold η_i^*.

Proposition 5. *In industry i, a firm would choose vertical integration if $\eta>\eta_i^*$, sourcing if $\eta<\eta_i^*$. The share of firms that choose vertical integration is $1-F\left(\eta_i^*\right)$.*

B Notation for the inter-firm matching market in the quantitative model

Similar to the labor market tightness ratio, the tightness ratios of inter-firm matching market is the ratio of the number of single firms in two sets. In the Nash equilibrium, firms only match with firms of same type. Thus there are two endogenously segmented markets. In sum, the model has two sets and two submarkets which give rise to four tightness ratios.

$$\begin{aligned}\tilde{\theta}^{jk}_{A,l,t} &= \frac{n^{j}_{A,l,t}}{n^{k}_{B,l,t}} \quad j,k \in \{H,L\}, \; l \in \{VI,SC\} \\ \tilde{\theta}^{kj}_{B,l,t} &= \frac{n^{k}_{B,l,t}}{n^{j}_{A,l,t}}\end{aligned}$$

where $n^{j}_{i,l,t}$ is the measure of single firms of type j in set i and plan to use inter-firm cooperation contract l. I use tilde to indicate the inter-firm tightness ratios.

Inter-firm matching rate is the probability of a single firm matching with a partner.

$$\begin{aligned}p^{jk}_{A,l,t} &= \frac{\tilde{M}\left(n^{j}_{A,l,t}, n^{k}_{B,l,t}\right)}{n^{j}_{A,l,t}} \quad j,k \in \{H,L\} \; l \in \{VI,SC\} \\ p^{kj}_{B,l,t} &= \frac{\tilde{M}\left(n^{k}_{A,l,t}, n^{j}_{B,l,t}\right)}{n^{k}_{B,l,t}}\end{aligned}$$

where $\tilde{M}\left(n^{j}_{A,l,t}, n^{k}_{B,l,t}\right)$ is matching functions for inter-firm matching which is assumed to be homogeneous of degree one. As the result, set i's cooperation matching rate $p^{jk}_{i,l,t}$ is an increasing function of inter-firm tightness ratio $\tilde{\theta}^{jk}_{i,l,t}$.

Furthermore, I the inter-firm matching function $\tilde{M}$ to be Cobb-Douglas, that is,

$$\tilde{M}\left(n^{j}_{A,l,t}, n^{k}_{B,l,t}\right) = \psi\left(n^{j}_{A,l,t}\right)^{1-\alpha_2}\left(n^{k}_{B,l,t}\right)^{\alpha_2}, \; j,K \in \{H,L\}, \; l \in \{VI,SC\}$$

C Transition rule of firms (employments) in the quantitative model

The transition rule of firms (or equivalently, employment) of each department indexed by type in sector A is governed by the following equations.

C.1 Firms with vertical integration (VI)

- H type single firm

$$\begin{aligned} n_{A,VI}^{H'} &= (1-\delta)\left[(1-p_{A,VI}^{H})(1-\rho_A^{HL})n_{A,VI}^{H} + (1-p_{A,VI}^{L})\rho_A^{LH}n_{A,VI}^{L}\right] \\ &+\frac{1}{2}\left(1-F\left(\eta^*\right)\right)\mu_A u_A \end{aligned} \tag{C.1}$$

where P_A^H and P_A^L are cooperation matching rate with $P_{A,VI}^H = \frac{\tilde{M}\left(n_{A,VI}^H, n_{B,VI}^H\right)}{n_{A,VI}^H}$ and $P_{A,VI}^L = \frac{\tilde{M}\left(n_{A,VI}^L, n_{B,VI}^L\right)}{n_{A,VI}^L}$

- L type single firm

$$\begin{aligned} n_{A,VI}^{L'} &= (1-\delta)\left[(1-p_{A,VI}^{L})(1-\rho_A^{LH})n_{A,VI}^{L} + (1-p_{A,VI}^{H})\rho_A^{LH}n_{A,VI}^{H}\right] \\ &+\frac{1}{2}\left(1-F\left(\eta^*\right)\right)\mu_A u_A \end{aligned} \tag{C.2}$$

- H type cooperative firm matched with H type partner

$$n_{A,VI}^{HH'} = (1-\delta)\left[p_{A,VI}^{H}n_{A,VI}^{H} + \sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lH}\rho_B^{qH}n_{A,VI}^{lq}\right] \tag{C.3}$$

- H type cooperative firm matched with L type partner

$$n_{A,VI}^{HL'} = (1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lH}\rho_B^{qL}n_{A,VI}^{lq}\right] \tag{C.4}$$

- L type cooperative firm matched with H type partner

$$n_{A,VI}^{LH'} = (1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qH}n_{A,VI}^{lq}\right] \tag{C.5}$$

- L type cooperative firm matched with L type partner

$$n_{A,VI}^{LL'} = (1-\delta)\left[p_{A,VI}^{L}n_{A,VI}^{L} + \sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qL}n_{A,VI}^{lq}\right] \tag{C.6}$$

The first two equations are flow motions for single firms. Take H type as an example. Existing H type single firms may flow out in three conditions: first, with probability δ an H type single firm can be destructed exogenously; Second, with probability p_A^H it is matched with another firm; Third, with probability ρ_A^{HL} the firm rotate to a L type single firm. At the same time, there are two kinds of inflow into H type: L type single firm becomes H type with probability ρ_A^{LH}; unemployed workers becomes H type employment (firm) with job finding rate μ_A. $(1-F(\eta^*))$ fraction of the new created firms choose to do vertical integration, and half of them draw an H type.

The last four equations are flow motion for cooperative firms. Take the HH type as an example. With probability δ existing firms are exogenously destructed. And with probability $\rho_A^{lH}\rho_B^{qH}$ an lq type firm becomes HH. Lastly, H type single firms match to another H type single firm and becomes HH with probability p_A^H.

C.2 Firms with sourcing (SC)

- H type single firm

$$\begin{aligned} n_{A,SC}^{H'} &= (1-\delta)\left[(1-p_{A,SC}^H)(1-\rho_A^{HL})n_{A,SC}^H + (1-p_{A,VI}^L)\rho_A^{LH} n_{A,SC}^L\right] \\ &+\frac{1}{2}F(\eta^*)\mu_A u_A + \nu(1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lH}\rho_B^{qL} n_{A,SC}^{lq}\right] \end{aligned} \quad (C.7)$$

where P_A^H and P_A^L are cooperation matching rate with $P_{A,SC}^H = \frac{\tilde{M}\left(n_{A,SC}^H, n_{B,SC}^H\right)}{n_{A,SC}^H}$ and $P_{A,SC}^L = \frac{\tilde{M}\left(n_{A,SC}^L, n_{B,SC}^L\right)}{n_{A,SC}^L}$

- L type single firm

$$\begin{aligned} n_{A,SC}^{L'} &= (1-\delta)\left[(1-p_{A,SC}^L)(1-\rho_A^{LH})n_{A,SC}^L + (1-p_{A,SC}^H)\rho_A^{LH} n_{A,SC}^H\right] \\ &+\frac{1}{2}F(\eta^*)\mu_A u_A + \nu(1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qH} n_{A,SC}^{lq}\right] \end{aligned} \quad (C.8)$$

- H type cooperative firm matched with H type partner

$$n_{A,SC}^{HH'} = (1-\delta)\left[p_{A,SC}^H n_{A,SC}^H + \sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lH}\rho_B^{qH} n_{A,SC}^{lq}\right] \quad (C.9)$$

- H type cooperative firm matched with L type partner

$$n_{A,SC}^{HL'} = (1-\nu)(1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lH}\rho_B^{qL}n_{A,SC}^{lq}\right] \tag{C.10}$$

- L type cooperative firm matched with H type partner

$$n_{A,SC}^{LH'} = (1-\nu)(1-\delta)\left[\sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qH}n_{A,SC}^{lq}\right] \tag{C.11}$$

- L type cooperative firm matched with L type partner

$$n_{A,SC}^{LL'} = (1-\delta)\left[p_{A,SC}^{L}n_{A,SC}^{L} + \sum_{l=\{H,L\}}\sum_{q=\{H,L\}}\rho_A^{lL}\rho_B^{qL}n_{A,SC}^{lq}\right] \tag{C.12}$$

The flow motion equations for SC firms are similar with the VI case, expect that mismatched firms have ν probability to sever the partnership and becomes a single firm.

C.3 Matrix form

Putting the equations to matrix form, I can stack the transition rule in matrix form.

For firms choosing VI

$$\begin{bmatrix} n_{A,VI}^{H'} \\ n_{A,VI}^{L'} \\ n_{A,VI}^{HH'} \\ n_{A,VI}^{HL'} \\ n_{A,VI}^{LH'} \\ n_{A,VI}^{LL'} \end{bmatrix} = \Phi_{VI} \times \begin{bmatrix} n_{A,VI}^{H} \\ n_{A,VI}^{L} \\ n_{A,VI}^{HH} \\ n_{A,VI}^{HL} \\ n_{A,VI}^{LH} \\ n_{A,VI}^{LL} \end{bmatrix} + \Xi_{VI} \times u_A \tag{C.13}$$

where

$$\Phi_{VI} = \begin{bmatrix} \left(1-p^{H}_{A,VI}\right)\left(1-\rho^{HL}_{A}\right) & \left(1-p^{L}_{A,VI}\right)\rho^{LH}_{A} & 0 & 0 & 0 & 0 \\ \left(1-p^{H}_{A,VI}\right)\rho^{LH}_{A} & \left(1-p^{L}_{A,VI}\right)\left(1-\rho^{LH}_{A}\right) & 0 & 0 & 0 & 0 \\ p^{H}_{A,VI} & 0 & & & & \\ 0 & 0 & \Pi^{T}_{A} & \otimes & \Pi^{T}_{B} & \\ 0 & 0 & & & & \\ 0 & p^{L}_{A,VI} & & & & \end{bmatrix} \tag{C.14}$$

$$\Xi_{VI} = \begin{bmatrix} \frac{1}{2}\left(1-F\left(\eta^{*}\right)\right)\mu(\theta_A) \\ \frac{1}{2}\left(1-F\left(\eta^{*}\right)\right)\mu(\theta_A) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{C.15}$$

In the Φ matrix, $\otimes$ is Kronecker product and Π_A and Π_B are Markov switching matrices that governs rotation of types.

Similarly, for firms choosing SC

$$\begin{bmatrix} n^{H'}_{A,SC} \\ n^{L'}_{A,SC} \\ n^{HH'}_{A,SC} \\ n^{HL'}_{A,SC} \\ n^{LH'}_{A,SC} \\ n^{LL'}_{A,SC} \end{bmatrix} = \Phi_{SC} \times \begin{bmatrix} n^{H}_{A,SC} \\ n^{L}_{A,SC} \\ n^{HH}_{A,SC} \\ n^{HL}_{A,SC} \\ n^{LH}_{A,SC} \\ n^{LL}_{A,SC} \end{bmatrix} + \Xi_{SC} \times u_A \tag{C.16}$$

where

$$\Xi_{SC} = \begin{bmatrix} \frac{1}{2}F\left(\eta^{*}\right)\mu(\theta_A) \\ \frac{1}{2}F\left(\eta^{*}\right)\mu(\theta_A) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{C.17}$$

, and

$$\Phi_{SC} = \begin{bmatrix} \left(1-p^{H}_{A,SC}\right)\left(1-\rho^{HL}_{A}\right) & \left(1-p^{L}_{A,SC}\right)\rho^{LH}_{A} & \nu\left(\Pi^{T}_{A}\otimes\Pi^{T}_{B}\right)_{(2,:)} \\ \left(1-p^{H}_{A,SC}\right)\rho^{LH}_{A} & \left(1-p^{L}_{A,SC}\right)\left(1-\rho^{LH}_{A}\right) & \nu\left(\Pi^{T}_{A}\otimes\Pi^{T}_{B}\right)_{(3,:)} \\ p^{H}_{A,SC} & 0 & \left(\Pi^{T}_{A}\otimes\Pi^{T}_{B}\right)_{(1,:)} \\ 0 & 0 & (1-\nu)\left(\Pi^{T}_{A}\otimes\Pi^{T}_{B}\right)_{(2,:)} \\ 0 & 0 & (1-\nu)\left(\Pi^{T}_{A}\otimes\Pi^{T}_{B}\right)_{(3,:)} \\ 0 & p^{L}_{A,SC} & \left(\Pi^{T}_{A}\otimes\Pi^{T}_{B}\right)_{(4,:)} \end{bmatrix} \tag{C.18}$$

where $\left(\Pi^{T}_{A}\otimes\Pi^{T}_{B}\right)_{(k,:)}$ denotes the kth row of $\left(\Pi^{T}_{A}\otimes\Pi^{T}_{B}\right)$ matrix.

Combine the VI and SC firms, the flow motion of employment is then described by

$$\underbrace{\begin{bmatrix} n^{H'}_{A,VI} \\ \vdots \\ n^{LL'}_{A,SC} \end{bmatrix}}_{12\times 1} = \underbrace{\Phi}_{12\times 12} \times \underbrace{\begin{bmatrix} n^{H}_{A,VI} \\ \vdots \\ n^{LL}_{A,SC} \end{bmatrix}}_{12\times 1} + \underbrace{\Xi}_{12\times 1} \times u_A \tag{C.19}$$

where

$$\Phi = \begin{bmatrix} \Phi_{SC} & \\ & \Phi_{VI} \end{bmatrix}$$

and

$$\Xi = \begin{bmatrix} \Xi_{SC} \\ \Xi_{VI} \end{bmatrix}$$

D Non-linear system of equations

D.1 Household's F.O.Cs

- Marginal utility of consumption

$$\xi_{c,t}\left(C_t - \Psi_c \bar{C}_{t-1}\right)^{-1} = \lambda_t \tag{D.1}$$

where λ is the Lagrange multiplier on the household's budget constraint. In equilibrium $C = \bar{C}$

- F.O.C of investment

$$\lambda_t = q_t \left[(1 - S_t) - S_t^{'} \xi_{I,t} \frac{I_t}{I_{t-1}} \right] + \beta E \left[q_{t+1} S_{t+1}^{'} \xi_{I,t+1} \frac{I_{t+1}^2}{I_t^2} \right] \tag{D.2}$$

where q is Tobin's q.

- F.O.C of capital stock

$$\lambda_t r_{k,t} = -q_t (1 - d) + \frac{q_{t-1}}{\beta} \tag{D.3}$$

D.2 Household's marginal value of employment and unemployment

- Household's marginal value of employment for each type of employee

$$\underbrace{\begin{bmatrix} V_{n_{i,VI}^H} \\ \vdots \\ V_{n_{i,SC}^{LL}} \end{bmatrix}}_{12\times 1} = -\xi_n + \lambda \underbrace{\begin{bmatrix} w_{i,VI}^H \\ \vdots \\ w_{i,SC}^{LL} \end{bmatrix}}_{12\times 1} + \beta \underbrace{\Phi^T}_{12\times 12} E \underbrace{\begin{bmatrix} V_{n_{i,VI}^H}^{'} \\ \vdots \\ V_{n_{i,SC}^{LL}}^{'} \end{bmatrix}}_{12\times 1} \tag{D.4}$$

  where Φ is defined in the last section and T denotes transpose of matrix.

- Equilibrium condition for perfect labor mobility and household's marginal value of unemployment.

$$V_{u_A} = V_{u_B} = V_u \tag{D.5}$$

where

$$\begin{aligned} V_{u_A} &= z \cdot \lambda + \beta (1 - \delta - \mu (\theta_A)) E \left[V_{u_A}^{'} \right] + \beta \underbrace{\Xi^T}_{1\times 12} E \underbrace{\begin{bmatrix} V_{n_{A,VI}^H}^{'} \\ \vdots \\ V_{n_{A,SC}^{LL}}^{'} \end{bmatrix}}_{6\times 1} \\ V_{u_B} &= z \cdot \lambda + \beta (1 - \delta - \mu (\theta_B)) E \left[V_{u_A}^{'} \right] + \beta \Xi^T \begin{bmatrix} V_{n_{B,VI}^H}^{'} \\ \vdots \\ V_{n_{B,SC}^{LL}}^{'} \end{bmatrix} \end{aligned}$$

With assumption of perfect labor mobility, the marginal value of unemployment should equalize across two sectors in any state of economy.

D.3 Firm's FOCs

- Firm's F.O.C. of capital input.

$$xz_i^H \left(\frac{k_{i,VI}^H}{xn_{i,VI}^H}\right)^{\alpha-1} = \cdots = xz_i^{LL} \left(\frac{k_{i,SC}^{LL}}{xn_{i,SC}^{LL}}\right)^{\alpha-1} = r_K \tag{D.6}$$

  Firm allocates capital to production departments to the point that *mpk* of each department equal to rental rate of capital.

- Firm's F.O.C. of vacancy posting.

$$\chi = \beta \int \left\{ \Xi_{\theta_A}^T \begin{bmatrix} E\left[\frac{\lambda'}{\lambda}\frac{\partial J_i'}{\partial n_{i,VI}^H(\eta)}\right] \\ \vdots \\ E\left[\frac{\lambda'}{\lambda}\frac{\partial J_i'}{\partial n_{i,SC}^{LL}(\eta)}\right] \end{bmatrix} \right\} dF(\eta) \tag{D.7}$$

  Firm chooses the tightness ratio so that its expected marginal value equals to vacancy posting cost.

D.4 Firm's envelope conditions

- Firm, number of employee in each department

$$\lambda \begin{bmatrix} \frac{\partial J_i}{\partial n_{i,VI}^H(\eta)} \\ \vdots \\ \frac{\partial J_i}{\partial n_{i,SC}^{LL}(\eta)} \end{bmatrix} = \lambda \begin{bmatrix} mpl_{i,VI}^H \\ \vdots \\ mpl_{i,SC}^{LL} \end{bmatrix} - \lambda \begin{bmatrix} w_{i,VI}^H \\ \vdots \\ w_{i,SC}^{LL} \end{bmatrix} + \beta \Phi^T \left(\tilde{\theta}, \Pi\right) E \left(\lambda' \begin{bmatrix} \frac{\partial V'}{\partial n_{i,VI}^H(\eta)} \\ \vdots \\ \frac{\partial V'}{\partial n_{i,SC}^{LL}(\eta)} \end{bmatrix} \right) \tag{D.8}$$

D.5 Total surplus

The definition of total surplus TS as

$$TS_{i,l}^{j}(\eta)=\lambda\frac{\partial J_i}{\partial n_{i,l}^{j}(\eta)}+\frac{\partial V}{\partial n_{i,l}^{j}(\eta)}-\frac{\partial V}{\partial u_i}$$

And with Nash Bargaining we have

$$\lambda\frac{\partial J_i}{\partial n_{i,l}^{j}(\eta)} = \tau TS_{i,l}^{j}(\eta) \tag{D.9}$$

$$\frac{\partial V}{\partial n_{i,l}^{j}(\eta)}-\frac{\partial V}{\partial u_i} = (1-\tau)\,TS_{i,l}^{j}(\eta) \tag{D.10}$$

By combining household's F.O.C., firm's F.O.C. and envelope conditions together one can get the following total surplus representation

$$\begin{aligned}\begin{bmatrix}TS_{i,l}^{H}(\eta)\\ TS_{i,l}^{L}(\eta)\end{bmatrix} &= \lambda\left(\begin{bmatrix}mpl_{i,l}^{H}\\ mpl_{i,l}^{L}\end{bmatrix}-z-\frac{(1-\delta)(1-\tau)\chi\theta_i}{\tau}\right)-\xi_n\\ &+\beta(1-\delta)E\begin{bmatrix}p_i^{H}TS_{i,l}^{HH}+\left(1-p_i^{H}\right)\left(\rho_i^{HH}TS_{i,l}^{H}(\eta)+\rho_i^{HL}TS_{i,l}^{L}(\eta)\right)\\ p_i^{L}TS_{i,l}^{LL}+\left(1-p_i^{L}\right)\left(\rho_i^{LH}TS_{i,l}^{H}(\eta)+\rho_i^{LL}TS_{i,l}^{L}(\eta)\right)\end{bmatrix}\end{aligned} \tag{D.11}$$

$$\begin{aligned}\begin{bmatrix}TS_{i,l}^{HH}(\eta)\\ TS_{i,l}^{HL}(\eta)\\ TS_{i,l}^{LH}(\eta)\\ TS_{i,l}^{LL}(\eta)\end{bmatrix} &= \lambda\left(\begin{bmatrix}mpl_{i,l}^{HH}\\ mpl_{i,l}^{HL}\\ mpl_{i,l}^{LH}\\ mpl_{i,l}^{LL}\end{bmatrix}-z-\frac{(1-\delta)(1-\tau)\chi\theta_i}{\tau}\right)-\xi_n\\ &+\beta(1-\delta)\left(\Pi_A^{T}\otimes\Pi_B^{T}\right)E\begin{bmatrix}TS_{i,l}^{HH'}(\eta)\\ TS_{i,l}^{HL'}(\eta)\\ TS_{i,l}^{LH'}(\eta)\\ TS_{i,l}^{LL'}(\eta)\end{bmatrix}\end{aligned} \tag{D.12}$$

And the free entry condition of the labor market i becomes

$$\lambda\chi = \beta\tau\int\left\{\Xi_{\theta_A}^{T}E\begin{bmatrix}TS_{i,VI}^{H}(\eta)\\ \vdots\\ TS_{i,SC}^{LL}(\eta)\end{bmatrix}\right\}dF(\eta) \tag{D.13}$$

D.6 The choice of contract and the free entry condition in the quantitative model

In this subsection, I show the determination of cooperation contract and the free entry condition.

A firm's first order condition with regard to the measure of vacancies v_i gives the free entry condition of the model

$$\frac{\chi}{f(\theta_i)} = E\left\{\frac{\lambda'}{\lambda}\int \max\left[\frac{\frac{\partial J_i}{\partial n^H_{i,SC}(\eta)}+\frac{\partial J_i}{\partial n^L_{i,SC}(\eta)}}{2}, \frac{\frac{\partial J_i}{\partial n^H_{i,VI}}+\frac{\partial J_i}{\partial n^L_{i,VI}}}{2}\right] dF(\eta)\right\} \quad \text{(D.14)}$$

where $J_{i,n^j_{i,l}}$ is a firm's marginal value of employment with type j, cooperation contract l, in set i.

A single firm with transaction cost η would choose VI if and only if

$$\frac{\partial J_i}{\partial n^H_{i,SC}(\eta)}+\frac{\partial J_i}{\partial n^L_{i,SC}(\eta)} < \frac{\partial J_i}{\partial n^H_{i,VI}}+\frac{\partial J_i}{\partial n^L_{i,VI}}$$

or equivalently

$$TS^H_{i,SC}(\eta)+TS^L_{i,SC}(\eta) < TS^H_{i,VI}+TS^L_{i,VI}$$

We can categorize firms' choice of contract with the following proposition

Proposition 6. *In industry i and in period t, a single firm would choose vertical integration if $\eta \geq \eta^*_{i,t}$, sourcing if $\eta < \eta^*_{i,t}$. The threshold is determined by*

$$\eta^*_t = \beta(1-\delta)\left(\rho_{A,t}+\rho_{B,t}\right)E_t\left[\tilde{TS}_{i,VI,t+1}-(1-\nu)\tilde{TS}_{i,SC,t+1}(\eta^*_t)\right]$$

with

$$\begin{aligned}\tilde{TS}_{i,SC} &\triangleq TS^{HH}_{i,SC}(\eta)+TS^{LL}_{i,SC}(\eta)-TS^{LH}_{i,SC}(\eta)-TS^{HL}_{i,SC}(\eta)\\ \tilde{TS}_{i,VI} &\triangleq TS^{HH}_{i,VI}+TS^{LL}_{i,VI}-TS^{LH}_{i,VI}-TS^{HL}_{i,VI}\end{aligned}$$

In the above proposition, as I proved in the appendix, both $\tilde{TS}_{i,SC}$ and $\tilde{TS}_{i,VI}$ are independent with η. We can rewrite free entry condition equation as

$$\chi = \tau\beta f(\theta_{i,t}) E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\begin{array}{c} \int_{-\infty}^{\eta_t^*} \frac{TS^H_{i,SC,t+1}(\eta)+TS^L_{i,SC,t+1}(\eta)}{2} dF(\eta) \\ + \\ (1-F(\eta_t^*)) \frac{TS^H_{i,VI,t+1}+TS^L_{i,VI,t+1}}{2} \end{array} \right] \right\} \tag{D.15}$$

The following result substantially simplifies the computation of the model.

Proposition 7. *The free entry condition of set i is*

$$\chi = \tau\beta f(\theta_{i,t}) E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{TS^H_{i,VI,t+1}+TS^L_{i,VI,t+1}}{2} + F(\eta_t^*) \Delta\hat{TS}_{i,t+1} \right] \right\}$$

with

$$\begin{aligned} \Delta\hat{TS}_{i,t} &= -\hat{\eta}_t + \\ &\quad \beta(1-\delta) E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\Delta\hat{TS}_{i,t+1} - (\rho_{A,t}+\rho_{B,t})(1-\nu)\tilde{TS}_{i,SC,t+1} + (\rho_{A,t}+\rho_{B,t})\tilde{TS}_{i,VI,t+1} \right] \right\} \\ \hat{\eta}_t &\triangleq \frac{\int_{-\infty}^{\eta_t^*} \eta \, dF(\eta)}{F(\eta_t^*)} \end{aligned}$$

E Proof of lemmas and propositions

E.1 Proof of results of section 2 and 3

In this subsection, I will use the following notations

$\tilde{J}_i = J_i^{HH} + J_i^{LL} - J_i^{HL} - J_i^{LH}$

$\tilde{z}_i = z_i^{HH} + z_i^{LL} - z_i^{HL} - z_i^{LH}$

$\bar{J}_i = \frac{J_i^{HH}+J_i^{LL}}{2}$

$\bar{z}_i = \frac{z_i^{HH}+z_i^{LL}}{2}$

$\tilde{J}_{i,l}(\eta) = J_{i,l}^{HH}(\eta) + J_{i,l}^{LL}(\eta) - J_{i,l}^{LH}(\eta) - J_{i,l}^{HL}(\eta)$

$i \in \{A,B\},\ l \in \{VI,SC\}$

As the two sets of firms are symmetric, to save space I will drop the subscript i in some cases.

Lemma 1

(1) Value function is strictly monotone if idiosyncratic productivity is strictly monotone.

(2) Value function is supermodular if and only if production function is supermodular.

Proof. (1) Use firms' value functions, we get

$$\begin{bmatrix} J_t^{HH} - J_t^{HL} \\ J_t^{LH} - J_t^{LL} \end{bmatrix} = \tau \cdot \begin{bmatrix} z^{HH} - z^{HL} \\ z^{LH} - z^{LL} \end{bmatrix} + \beta (1-\delta) \begin{bmatrix} \rho^{HH} & \rho^{HL} \\ \rho^{LH} & \rho^{LL} \end{bmatrix} \begin{bmatrix} J_t^{HH} - J_t^{HL} \\ J_t^{LH} - J_t^{LL} \end{bmatrix}$$

$$\Rightarrow$$

$$\tau \cdot \begin{bmatrix} z^{HH} - z^{HL} \\ z^{LH} - z^{LL} \end{bmatrix} = \begin{bmatrix} 1-\beta(1-\delta)\rho^{HH} & -\beta(1-\delta)\rho^{HL} \\ -\beta(1-\delta)\rho^{LH} & 1-\beta(1-\delta)\rho^{LL} \end{bmatrix} \begin{bmatrix} J_t^{HH} - J_t^{HL} \\ J_t^{LH} - J_t^{LL} \end{bmatrix}$$

$$\Rightarrow$$

$$\begin{bmatrix} J_t^{HH} - J_t^{HL} \\ J_t^{LH} - J_t^{LL} \end{bmatrix} = \frac{\tau \cdot \begin{bmatrix} 1-\beta(1-\delta)\rho^{LL} & \beta(1-\delta)\rho^{HL} \\ \beta(1-\delta)\rho^{LH} & 1-\beta(1-\delta)\rho^{HH} \end{bmatrix} \begin{bmatrix} z^{HH} - z^{HL} \\ z^{LH} - z^{LL} \end{bmatrix}}{(1-\beta(1-\delta)\rho^{HH})(1-\beta(1-\delta)\rho^{LL}) + \beta^2 (1-\delta)^2 \rho^{HL}\rho^{LH}}$$

We immediately get that $\begin{bmatrix} J_t^{HH} - J_t^{HL} \\ J_t^{LH} - J_t^{LL} \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ if $\begin{bmatrix} z^{HH} - z^{HL} \\ z^{LH} - z^{LL} \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

With the same method we can prove that $\begin{bmatrix} J_t^{HH} - J_t^{LH} \\ J_t^{HL} - J_t^{LL} \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ if $\begin{bmatrix} z^{HH} - z^{LH} \\ z^{HL} - z^{LL} \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(2) It is to show that $\tilde{J} > 0$ if and only if $\tilde{z} > 0$. Use the value functions of firms equation 2 and equation refE2, we get

$$\begin{aligned} \tilde{J}_t &= \tau \cdot x_t \cdot z_t^{HH} + \beta(1-\delta) E_t \left(\rho_t^{HH} J_{t+1}^{HH} + \rho_t^{HL} J_{t+1}^{LH} \right) \\ &- \left[\tau \cdot x_t \cdot z_t^{HL} + \beta(1-\delta) E_t \left(\rho_t^{HH} J_{t+1}^{HL} + \rho_t^{HL} J_{t+1}^{LL} \right) \right] \\ &+ \tau \cdot x_t \cdot z_t^{LL} + \beta(1-\delta) E_t \left(\rho_t^{LH} J_{t+1}^{HL} + \rho_t^{LL} J_{t+1}^{LL} \right) \\ &- \left[\tau \cdot x_t \cdot z_t^{LH} + \beta(1-\delta) E_t \left(\rho_t^{LH} J_{t+1}^{HH} + \rho_t^{LL} J_{t+1}^{LH} \right) \right] \end{aligned}$$

Rearranging the terms, we get

$$\begin{aligned} \tilde{J}_t &= \tau \cdot x_t \left(z_t^{HH} + z_t^{LL} - z_t^{HL} - z_t^{LH} \right) \\ &+ \beta(1-\delta) \left(\rho_t^{HH} - \rho_t^{LH} \right) E_t \left(J_{t+1}^{HH} \right) \\ &+ \beta(1-\delta) \left(\rho_t^{LL} - \rho_t^{HL} \right) E_t \left(J_{t+1}^{LL} \right) \\ &+ \beta(1-\delta) \left(\rho_t^{LH} - \rho_t^{HH} \right) E_t \left(J_{t+1}^{HL} \right) \\ &+ \beta(1-\delta) \left(\rho_t^{HL} - \rho_t^{LL} \right) E_t \left(J_{t+1}^{LH} \right) \end{aligned}$$

Notice that the Markov switching matrix is symmetric, we have

$$\begin{aligned}\rho_t^{HH}-\rho_t^{LH} &= \rho_t^{LL}-\rho_t^{HL},\ 1-2\rho_t^{HL}=1-2\rho_t^{HL}\\ \rho_t^{LH}-\rho_t^{HH} &= -\left(1-2\rho_t^{HL}\right),\ \rho_t^{HL}-\rho_t^{LL}=-\left(1-2\rho_t^{HL}\right)\end{aligned}$$

Thus we have

$$\tilde{J}_t=\tau\cdot x_t\tilde{z}_t+\beta\left(1-2\rho_t^{HL}\right)(1-\delta)E_t\left(\tilde{J}_{t+1}\right) \tag{E.1}$$

In the steady state, we have

$$\tilde{J}=\frac{\tau\cdot x\cdot\tilde{z}}{1-\beta\left(1-2\rho^{HL}\right)(1-\delta)}$$

Since $1-\beta\left(1-2\rho^{HL}\right)(1-\delta)>0$, $\tilde{J}>0$ if and only if $\tilde{z}>0$.

□

Proposition 2

If production function is strictly monotone, and there is firm inter-connectivity, a persistent increase in the rotation rate of one set, with persistence bounded below $|\psi|<1-\rho^{HL}-\rho^{LH}$, state would lead to

1. a decrease in the equilibrium tightness ratio of both sets
2. a decrease in the average productivity of both sets in the next period

if and only if the production function is supermodular.

Proof. I will show when $\tilde{z}>0$, θ is decreasing with ρ^{HL}. Notice that θ is increasing in $\bar{J}$ according to free entry condition, we only need to show $\bar{J}$ is decreasing with ρ^{HL}

Fixing aggregate productivity x to one, we have the following equations

$$\bar{J}_t = \bar{z}+\beta(1-\delta)\left[E_t\left(\bar{J}_{t+1}\right)-\frac{\rho_t^{HL}E_t\left(\tilde{J}_{t+1}\right)}{2}\right] \tag{E.2}$$

$$\tilde{J}_t = \tilde{z}+\beta(1-\delta)\left(1-2\rho_t^{HL}\right)E_t\left(\tilde{J}_{t+1}\right) \tag{E.3}$$

$$\rho_t^{HL} = \psi\rho_{t-1}^{HL}+\varepsilon_t \tag{E.4}$$

Denote $F_{t,t+s}=\frac{1}{2}E_t\left(\rho_{t+s}^{HL}\tilde{J}_{t+s+1}\right)$, the first equation becomes

$$\bar{J}_t=\frac{\bar{z}}{1-\beta(1-\delta)}-\sum_{s=0}^{\infty}F_{t,t+s}$$

It suffices to show that $\frac{dF_{t,t+s}}{d\varepsilon_t} > 0$ for $t > 0 \quad s \geq 0$. Here I prove the inequality for $s = 0$, the cases for $s > 0$ are similar.

$$\begin{aligned}
\frac{dF_{t,t}}{d\varepsilon_t} &= \frac{d\left[\rho_t^{HL} E_t\left(\tilde{J}_{t+1}\right)\right]}{d\rho_t^{HL}} \\
&= E_t\left(\tilde{J}_{t+1}\right) + \rho_t^{HL} \frac{E_t\left(\tilde{J}_{t+1}\right)}{d\rho_t^{HL}}
\end{aligned}$$

According to equation E.3, we have

$$E_t\left(\tilde{J}_{t+1}\right) = \tilde{z} \cdot \sum_{s=0}^{\infty} E_t\left[\beta^s (1-\delta)^s \prod_{k=0}^{s} \left(1 - 2\rho_{t+k+1}^{HL}\right)\right]$$

Therefore

$$\begin{aligned}
\frac{E_t\left(\tilde{J}_{t+1}\right)}{d\rho_t^{HL}} &= \tilde{z} \cdot \sum_{s=0}^{\infty} E_t\left\{\left[\beta^s (1-\delta)^s \prod_{k=0}^{s} \left(1 - 2\rho_{t+k+1}^{HL}\right)\right] \cdot \sum_{k=0}^{s} \left(-\frac{2}{1 - 2\rho_{t+k+1}^{HL}} \cdot \frac{d\rho_{t+k+1}^{HL}}{d\rho_t^{HL}}\right)\right\} \\
&> \tilde{z} \cdot \sum_{s=0}^{\infty} E_t\left\{\left[\beta^s (1-\delta)^s \prod_{k=0}^{s} \left(1 - 2\rho_{t+k+1}^{HL}\right)\right] \cdot \sum_{k=0}^{s} \left(-2 \cdot \frac{1 - \psi^k}{1 - \psi}\right)\right\} \\
&> -2 \cdot \tilde{z} \cdot \sum_{s=0}^{\infty} E_t\left[\frac{\beta^s (1-\delta)^s \prod_{k=0}^{s} \left(1 - 2\rho_{t+k+1}^{HL}\right)}{1 - \psi}\right] \\
&= -\frac{2}{1-\psi} \cdot E_t\left(\tilde{J}_{t+1}\right)
\end{aligned}$$

When $\frac{1-\psi}{\rho^{HL}} > 2$, we immediately get

$$\frac{dF_{t,t}}{d\varepsilon_t} > \left(1 - \frac{2\rho^{HL}}{1-\psi}\right) E_t\left(\tilde{J}_{t+1}\right) > 0$$

□

Proposition

The threshold level of transaction cost of industry i exists and is uniquely determined by

$$\tau \cdot \eta_i^* = \beta (1-\delta) g\left(\rho_j^{HL}, \nu\right) \cdot \left(z_i^{HH} + z_i^{LL} - z_i^{HL} - z_i^{LH}\right)$$

with $\frac{\partial g\left(\rho_j^{HL}, \nu\right)}{\partial \rho_j^{HL}} > 0$ and $\frac{\partial g\left(\rho_j^{HL}, \nu\right)}{\partial \nu} > 0$

Proof. The threshold level of transaction cost is determined by

$$J_{i,VI}^{HH}+J_{i,VI}^{LL}=J_{i,SC}^{HH}(\eta_i^*)+J_{i,SC}^{LL}(\eta_i^*) \tag{E.5}$$

The LHS of equation E.5 can be derived as

$$J_{i,VI}^{HH}+J_{i,VI}^{LL}=\tau\left(z_i^{HH}+z_i^{LL}\right)+\beta(1-\delta)E_t\begin{bmatrix}J_{i,VI}^{HH}+J_{i,VI}^{LL}\\ -\rho_t\tilde{J}_{i,VI}\end{bmatrix} \tag{E.6}$$

The RHS of equation E.5 can be derived as

$$J_{i,SC}^{HH}(\eta_i^*)+J_{i,SC}^{LL}(\eta_i^*)=\tau\left(z_i^{HH}+z_i^{LL}-2\eta_i^*\right)+\beta(1-\delta)E_t\begin{bmatrix}J_{i,SC}^{HH}(\eta_i^*)+J_{i,SC}^{LL}(\eta_i^*)\\ -\rho_t(1-\nu)\tilde{J}_{i,SC}(\eta_i^*)\end{bmatrix} \tag{E.7}$$

Subtract equation E.6 from E.7, we get

$$2\tau\eta_i^*=\beta(1-\delta)\rho_t\left[\tilde{J}_{i,VI}-(1-\nu)\tilde{J}_{i,SC}(\eta_i^*)\right] \tag{E.8}$$

I first present a useful lemma.

Lemma. *Mismatch loss for firms choosing sourcing is constant within industry, that is*

$$\tilde{J}_{i,SC}(\eta_i^*)=\tilde{J}_{i,SC}$$

for $\eta\sim\mathcal{N}\left(\bar{\eta}_i,\sigma_i^2\right)$

Proof. By plugging value functions into the above equation and using the method of deriving equation E.1, we get

$$\tilde{J}_{SC,t}(\eta)=\tau\tilde{z}_t+\beta(1-\delta)\left[1-\left(\rho_t^{HL}+\rho_t^{LH}\right)(1-\nu)\right]E_t\left(\tilde{J}_{SC,t}(\eta)\right)$$

As shown in the above equation, mismatch loss $J_{S,t}\tilde{}(\eta)$ is independent of contraction cost η, and I will drop the η term in the following proof.

□

With the above lemma, we immediately get that in the steady state,

$$\tilde{J}_{SC}=\frac{\tau\tilde{z}}{1-\beta(1-\delta)[1-2\rho(1-\nu)]} \tag{E.9}$$

, where $\rho \triangleq \rho^{HL} = \rho^{LH}$.

For firms who choose vertical integration, the mismatch loss is same as the simple model, and we have

$$\tilde{J}_{VI} = \frac{\tau \tilde{z}}{1-\beta(1-\delta)(1-2\rho)} \tag{E.10}$$

Plug equations E.9 and E.10 into equation E.8, we get

$$\begin{aligned}
\eta_i^* &= \frac{\beta(1-\delta)\tilde{z}}{2}\left[\frac{\rho}{1-\beta(1-\delta)(1-2\rho)} - \frac{(1-\nu)\rho}{1-\beta(1-\delta)[1-2\rho(1-\nu)]}\right] \\
&= \frac{\beta(1-\delta)\tilde{z}}{2} \cdot \frac{\rho\nu[1-\beta(1-\delta)]}{[1-\beta(1-2\rho)] \cdot \{1-\beta(1-\delta)[1-2\rho(1-\nu)]\}}
\end{aligned}$$

Denote

$$g(\rho,\nu) = \frac{\rho\nu}{2[1-\beta(1-2\rho)] \cdot \{1-\beta(1-\delta)[1-2\rho(1-\nu)]\}}$$

Therefore,

$$\eta_i^* = \beta(1-\delta)\tilde{z}g(\rho,\nu)$$

With tedious algebra, we can show that

$$\frac{\partial g(\rho,\nu)}{\partial \rho} = \frac{\nu\left[1-\beta(1-\delta)+\beta^2(1-\delta)^2\left(1-4(1-\nu)\rho^2\right)\right]}{[1-\beta(1-2\rho)]^2 \cdot \{1-\beta(1-\delta)[1-2\rho(1-\nu)]\}^2}$$

and

$$\frac{\partial g(\rho,\nu)}{\partial \nu} = \frac{\rho\left[1-\beta(1-\delta)(1-4\rho)+\beta^2(1-\delta)^2\left(1-4\rho+4\rho^2\right)\right]}{[1-\beta(1-2\rho)]^2 \cdot \{1-\beta(1-\delta)[1-2\rho(1-\nu)]\}^2}$$

In the data, ρ is never larger than 0.2, hence we have

$$\frac{\partial g(\rho,\nu)}{\partial \rho} > 0$$

and

$$\frac{\partial g(\rho,\nu)}{\partial \nu} > 0$$

□

Proposition 5

In industry i, a firm would choose vertical integration if $\eta > \eta_i^*$, sourcing if $\eta < \eta_i^*$. The share of firms that choose vertical integration is $1 - F(\eta_i^*)$.

Proof. From a firm' value functions, it can be shown that

$$\begin{array}{ccc} \eta & \lesseqgtr & \eta^* = \dfrac{\nu\beta(1-\delta)\rho_j^{HL}\cdot\tilde{J}_i}{\tau} \\ & \Leftrightarrow & \\ \dfrac{1}{2}\left(J_{i,VI}^{HH}+J_{i,VI}^{LL}\right) & \lesseqgtr & \dfrac{1}{2}\left[J_{i,SC}^{HH}(\eta)+J_{i,SC}^{LL}(\eta)\right] \end{array}$$

□

In the following proofs, I use the following notations

$\tilde{TS}_{i,l}(\eta) = TS_{i,l}^{HH}(\eta) + TS_{i,l}^{LL}(\eta) - TS_{i,l}^{LH}(\eta) - TS_{i,l}^{HL}(\eta)$

$\tilde{z}_i = \left(z_i^{HH}\right)^{\frac{1}{1-\alpha}} - \left(z_i^{HL}\right)^{\frac{1}{1-\alpha}} - \left(z_i^{LH}\right)^{\frac{1}{1-\alpha}} + \left(z_i^{LL}\right)^{\frac{1}{1-\alpha}}$

$\bar{z}_i = \frac{\left(z_i^{HH}\right)^{\frac{1}{1-\alpha}} + \left(z_i^{LL}\right)^{\frac{1}{1-\alpha}}}{2}$

$i \in \{A, B\},\ l \in \{VI, SC\}$

As sets A and B are symmetric, to save space I will drop the subscript i in some cases.

Proposition 4

The positive assortative matching is Nash equilibrium if

$$\left(\frac{\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}}{\frac{\partial J_A}{\partial n_{A,l}^{HL}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}}\right)^{\frac{1}{\alpha_2}} \times \left(\frac{\frac{\partial J_B}{\partial n_{B,l}^{LH}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}}{\frac{\partial J_B}{\partial n_{B,l}^{LL}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}}\right)^{\frac{1}{1-\alpha_2}} > 1$$

and

$$\left(\frac{\frac{\partial J_B}{\partial n_{B,l}^{HH}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{H}(\eta)}}{\frac{\partial J_B}{\partial n_{B,l}^{HL}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{H}(\eta)}}\right)^{\frac{1}{1-\alpha_2}} \times \left(\frac{\frac{\partial J_A}{\partial n_{A,l}^{LH}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{L}(\eta)}}{\frac{\partial J_A}{\partial n_{A,l}^{LL}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{L}(\eta)}}\right)^{\frac{1}{\alpha_2}} > 1$$

Proof. I only need to show that no H type firms want to search for L type partners. I will show this for the firms in set A. Firms in set B have the same result.

For the H type single firm in set A choosing cooperation contract l, the expected gain of searching for H type partner is

$$\frac{\tilde{M}\left(n_{A,l}^{H}, n_{B,l}^{H}\right)}{n_{A,l}^{H}}\left(\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right)$$

where $\frac{\tilde{M}\left(n_{A,l}^{H}, n_{B,l}^{H}\right)}{n_{A,l}^{H}}$ is the probability of matching with a H type partner; $\left(\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right)$ is the marginal benefit conditional on matching with a H type partner.

Similarly, for the L type single firm in set B, the expected gain of searching for L type partner is

$$\frac{\tilde{M}\left(n_{A,l}^{L}, n_{B,l}^{L}\right)}{n_{B,l}^{L}}\left(\frac{\partial J_B}{\partial n_{B,l}^{LL}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}\right)$$

The Nash equilibrium is that firms search for same type partner only. Now assuming that a infinitesimal measure Δ_A^H of H firms in set A declare that they are searching for L type partner in set B. Knowing this, a certain measure of L type firms in set B will attempt to match with those H type firms; the measure is pinned down by the condition under which they are indifferent between matching with H and L. Specifically, the measure of L type firms in set B who commit deviating, Δ_B^L, is determined by

$$\frac{\tilde{M}\left(\Delta_A^H, \Delta_B^L\right)}{\Delta_B^L}\left(\frac{\partial J_B}{\partial n_{B,l}^{LH}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}\right) = \frac{\tilde{M}\left(n_{A,l}^{L}, n_{B,l}^{L}\right)}{n_{B,l}^{L}}\left(\frac{\partial J_B}{\partial n_{B,l}^{LL}(\eta)} - \frac{\partial J_B}{\partial n_{B,l}^{L}(\eta)}\right) \quad \text{(E.11)}$$

Given the measure of L type firms in set B attempting to match with H type firms in set A, the H type firms in set A who search for L type partner expect to gain

$$\frac{\tilde{M}\left(\Delta_A^H, \Delta_B^L\right)}{\Delta_A^H}\left(\frac{\partial J_A}{\partial n_{A,l}^{HL}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right)$$

Now I want to show that the above expected gain is less than the expected gain of searching for H type partner; that is,

$$\frac{\tilde{M}\left(\Delta_A^H, \Delta_B^L\right)}{\Delta_A^H}\left(\frac{\partial J_A}{\partial n_{A,l}^{HL}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right) < \frac{\tilde{M}\left(n_{A,l}^{H}, n_{B,l}^{H}\right)}{n_{A,l}^{H}}\left(\frac{\partial J_A}{\partial n_{A,l}^{HH}(\eta)} - \frac{\partial J_A}{\partial n_{A,l}^{H}(\eta)}\right) \quad \text{(E.12)}$$

As $n^H_{A,l} = n^L_{A,l} = n^H_{B,l} = n^L_{B,l}$ in PAM equilibrium. It is easy to show that $\frac{\tilde{M}\left(n^L_{A,l}, n^L_{B,l}\right)}{n^L_{B,l}} = \frac{\tilde{M}\left(n^H_{A,l}, n^H_{B,l}\right)}{n^H_{A,l}} = \psi$, and denote $\theta_{dev} = \frac{\Delta^L_B}{\Delta^H_A}$, we can rewrite equation E.11 and inequation E.12 as

$$\begin{aligned}
\psi\left(\theta_{dev}\right)^{\alpha_2-1}\left(\frac{\partial J_B}{\partial n^{LH}_{B,l}(\eta)} - \frac{\partial J_B}{\partial n^L_{B,l}(\eta)}\right) &= \psi\left(\frac{\partial J_B}{\partial n^{LL}_{B,l}(\eta)} - \frac{\partial J_B}{\partial n^L_{B,l}(\eta)}\right) \\
\psi\left(\theta_{dev}\right)^{\alpha_2}\left(\frac{\partial J_A}{\partial n^{HL}_{A,l}(\eta)} - \frac{\partial J_A}{\partial n^H_{A,l}(\eta)}\right) &< \psi\left(\frac{\partial J_A}{\partial n^{HH}_{A,l}(\eta)} - \frac{\partial J_A}{\partial n^H_{A,l}(\eta)}\right)
\end{aligned}$$

which is equivalent with

$$\begin{aligned}
\theta_{dev} &= \left(\frac{\frac{\partial J_B}{\partial n^{LL}_{B,l}(\eta)} - \frac{\partial J_B}{\partial n^L_{B,l}(\eta)}}{\frac{\partial J_B}{\partial n^{LH}_{B,l}(\eta)} - \frac{\partial J_B}{\partial n^L_{B,l}(\eta)}}\right)^{\frac{1}{\alpha_2-1}} \\
\theta_{dev} &< \left(\frac{\frac{\partial J_A}{\partial n^{HH}_{A,l}(\eta)} - \frac{\partial J_A}{\partial n^H_{A,l}(\eta)}}{\frac{\partial J_A}{\partial n^{HL}_{A,l}(\eta)} - \frac{\partial J_A}{\partial n^H_{A,l}(\eta)}}\right)^{\frac{1}{\alpha_2}}
\end{aligned}$$

We immediately see that the inequation of proposition implies the above condition.

□

Proposition 6

In industry i and in period t, a single firm would choose vertical integration if $\eta \geq \eta^*_{i,t}$, sourcing if $\eta < \eta^*_{i,t}$. The threshold is determined by

$$\eta^*_t = \beta(1-\delta)\left(\rho_{A,t} + \rho_{B,t}\right)E_t\left[\tilde{TS}_{i,VI,t+1} - (1-\nu)\tilde{TS}_{i,SC,t+1}\left(\eta^*_t\right)\right]$$

Proof. For firms choosing VI, it is easy to show that

$$\frac{TS^{HH}_{VI,t} + TS^{LL}_{VI,t}}{2} = \bar{z}_t + \frac{1}{2}\beta(1-\delta)E_t\begin{bmatrix} TS^{HH}_{VI,t} + TS^{LL}_{VI,t} \\ -\rho_t\tilde{TS}_{VI,t+1}\end{bmatrix}$$

Also, it can be shown that

$$\tilde{TS}_{VI,t} = \tilde{z}_t + \beta(1-\delta)\left[1 - 2\left(\rho_{A,t} + \rho_{B,t}\right)\right]E_t\left(\tilde{TS}_{VI,t+1}\right)$$

For firms choosing SC

$$\frac{TS^{HH}_{SC,t}(\eta)+TS^{LL}_{SC,t}(\eta)}{2}=(\bar{z}_t-\eta_t)+\frac{1}{2}\beta(1-\delta)E_t\begin{bmatrix}TS^{HH}_{SC,t+1}(\eta)+TS^{LL}_{SC,t+1}(\eta)\\-(\rho_{A,t}+\rho_{B,t})(1-\nu)\tilde{TS}_{SC,t+1}\end{bmatrix}$$

It can be shown that

$$\tilde{TS}_{SC,t}=\tilde{z}+\beta(1-\delta)\left[1-2(\rho_{A,t}+\rho_{B,t})(1-\nu)\right]E_t\left(\tilde{TS}_{SC,t+1}\right)$$

The threshold is determined by

$$TS^{H}_{SC,t}(\eta)+TS^{L}_{SC,t}(\eta)=TS^{H}_{VI,t}+TS^{L}_{VI,t} \tag{E.13}$$

or equivalently

$$\begin{aligned}&p^{HH}_{SC,t}E_t\left(TS^{HH}_{SC,t+1}\right)+\left(1-p^{HH}_{SC,t}\right)E_t\left(TS^{H}_{SC,t+1}\right)+p^{LL}_{SC,t}E_t\left(TS^{LL}_{SC,t+1}\right)+\left(1-p^{LL}_{SC,t}\right)E_t\left(TS^{L}_{SC,t+1}\right)\\&=p^{HH}_{VI,t}E_t\left(TS^{HH}_{VI,t+1}\right)+\left(1-p^{HH}_{VI,t}\right)E_t\left(TS^{H}_{VI,t+1}\right)+p^{LL}_{VI,t}E_t\left(TS^{LL}_{VI,t+1}\right)+\left(1-p^{LL}_{VI,t}\right)E_t\left(TS^{L}_{VI,t+1}\right)\end{aligned}$$

In PAM equilibrium, $p^{HH}_{SC,t}=p^{LL}_{SC,t}=p^{HH}_{VI,t}=p^{LL}_{VI,t}$

To have analytical solution, I make the assumption that

$$E_t\left(TS^{j}_{l,t+1}(\eta)\right)\;\approx\;TS^{j}_{l,t}(\eta)$$

Equation E.13 can be approximated by

$$TS^{HH}_{SC,t}(\eta)+TS^{LL}_{SC,t}(\eta)=TS^{HH}_{VI,t}+TS^{LL}_{VI,t} \tag{E.14}$$

By solving equation E.14 with the results above, the threshold of transaction cost is determined by

$$\eta^*_t=\beta(1-\delta)E_t\left[(\rho_{A,t}+\rho_{B,t})\tilde{TS}_{VI,t+1}-(1-\nu)(\rho_{A,t}+\rho_{B,t})\tilde{TS}_{SC,t+1}\right]$$

□

F Data

Data for section 1

I obtain annual firm-level balance sheet information from Compustat for North America. My base Compustat sample covers the period 1960-2013 for 112 3-digit industries and consists of 31069 publicly-traded firms who have NAICS code. I define profit as Earnings before Interest, Taxes and Amortization (Ebita). Profit margin is defined as the ratio of Ebida to sales. For each two consecutive years, I keep the panel balanced by dropping the delisted and enlisted firms in the two years.[16] In the balanced panel for each two consecutive years, each firm-year observation is categorized into H type or L type according to its position in the profit/profit margin distribution of 3-digit NAICS industry the firm belongs to: a firm is H type if its annual profit is above the median; L type if below median. A rotation occurs when a firm switches its type in consecutive years. Rotation rate is the ratio of number of rotation to the number of firms.

I obtain industry value added series constructed by BEA, which are used for the weights to construct the aggregate rotation rate. Yearly industry value added is available only for 1-digit industries (sector), while value added for 2-digit industries are only available every five years in Input-Output (I-O) Accounts Data. I impute value added of 2 digit industries by treating their shares of value added within each 1-digit industry as constant overtime. In particular, value added of industry i in sector j is

$$Value\ added_{i,j,t} = Value\ added_{j,t} \times S_{i,j}$$

where $S_{i,j}$ is the share of value added of industry i in sector j computed from 2007 Input-Output (I-O) table constructed by BEA.

For the national unemployment rate, I use annual series of aggregate unemployment rate from 1970 to 2013, obtained from Current Employment Statistics (CES). I construct industry employment growth from the annual series of full-time equivalent employees by industry from 1998 to 2013[17], obtained from National Income and Product Accounts (NIPAs) con-

[16] Most of enlist and delist do not carry information on turbulence of firms' rankings. While the delist due to bankruptcy and insolvency may represent a permanent rotation that is not included in my measurement, it comprise less than 3 percents of delisted firms, which is quantitatively insignificant for my analysis.

[17] Starting from 1998, BEA changed the classification of industries, hence I only use the series after that.

structed by BEA. In the industry panel regression, I use the private and non-farm 3 digit NAICS industries which are available in both Compustat and NIPA, and have more than 8 firms in Compustat over the sample periods.

Data for section 4

For the estimation of the DSGE model, I use quarterly observations on eight data series from 1969 Q1 to 2013 Q4: aggregate unemployment, aggregate job openings rate, growth rate of real consumption per capita, growth rate of real investment per capita, growth rate of real per-hour wage, real interest rate, and rotation rate for the two sectors.

The benchmark model has two sets/sectors. To conduct estimation, I need to construct empirical counterpart of the two sectors. To do so, I categorize the two-digit NAICS industries into two categories: industries that are more production related, including Mining and logging, Construction, Manufacturing, and industries that are more productive service related, including Trade transportation and utilities, Information, Financial activities, Professional and business services. The former category consists 19 percent of non-government employment while the latter consists 48 percent. Four industries which are inappropriate to fit into either category are dropped: Education and health services, Leisure and hospitality, Other services, and Government. I construct the two annual series of sectoral rotation rates by averaging rotation rates at the 3 digit NAICS industry level weighted by their valued added. Then I interpolate them into quarterly series.

For the other observable variables, I use quarterly aggregate unemployment rate obtained from Current Employment Statistics (CES) and quarterly job openings obtained from HWOL (Barnichon (2010)). Real interest rate series are constructed by NY Fed. The per capita GDP growth rate, per capital investment growth rate, per capita consumption growth rate, and the wage growth rate are all extracted and constructed from National Income and Product Accounts (NIPAs).

G The selection of prior for the parameters

DSGE parameters

For parameters that are present in common DSGE studies, I keep priors close to previous work (e.g. Justiniano et al. (2011)). In particular, for investment adjustment cost, I choose

Gamma distributions with a mean of 4 and a standard deviation of 2. Prior for habit persistence is a distribution with a mean 0.5 and standard deviation 0.1. Prior for trend productivity growth rate is a Normal distribution with mean 0.004 and standard deviation 0.001.

I follow the literature in choosing rather diffuse priors for the structural shock processes. The priors for the autocorrelation parameters are Beta distributions with mean 0.5 and standard deviation 0.1. The priors for the standard deviations of shocks are Inverse Gamma distributions: priors for standard deviations of shocks have mean 0.005 and standard deviation 0.2.

Following Ilut and Schneider (2014), the priors for the standard deviation of observables are set in the following way: for an observable W with unconditional standard deviation σ_W, the prior for the standard deviation of measurement error on W is an Inverse Gamma distribution with mean $0.1\sigma_W$ and standard deviation $0.4\sigma_W$. Therefore, at the prior mean, measurement error would explain 1% fluctuation in W; at one standard deviation, it would explain 16% fluctuations in W.

Priors for the productivities

Since I have calibrated z_i^{HH} and z_i^{LL}, it remains to estimate z_i^{HL}, z_i^{LH}, z_i^{H} and z_i^{L}. I set their priors as Beta distributions. For single firms, I set the prior mean of z_i^{H} and z_i^{L} to 0.5 and 0.2, 50 percent of z_i^{HH} and z_i^{LL}. That is to say, at the prior mean, finding a cooperation can double productivity. Priors standard deviation of productivities are set to 0.1.

The prior mean of z_A^{HL} and z_B^{HL} is set to be 0.6, which means that cooperating with an L type partner can improve H type's productivity by 20%. I impose that in any inter-firm match partners split output evenly, hence $z_A^{HL} = z_B^{LH}$ and $z_A^{LH} = z_B^{HL}$.

Priors for the parameters of the labor market and the cooperation contract

For most labor market parameters, I choose fairly dispersed priors and set the prior mean based on calibration studies. I set prior for labor disutility ξ_n to Beta distribution with mean 0.2 and standard deviation 0.1, combining with unemployment insurance[18] implies that unemployment benefit is 45% of average wage when the model is computed at the prior mean.

I set the prior for labor market matching efficiency to Beta distribution with a mean of 0.7 and a standard deviation of 0.1, so that steady-state unemployment rate is 5% at the prior mean. I don't have information to target the range of inter-firm matching efficiency,

[18]Labor disutility needs to be adjusted by marginal utility to be comparable with monetary variables such as wage and unemployment insurance.

so I set its prior to have the same mean as labor market matching efficiency but with higher dispersion. For the bargaining share of firm τ, calibration studies use a wide range of values centering around 0.5. Therefore, I set a beta prior around 0.5. For the labor market matching elasticity α_1, instead of estimating it, I impose $\alpha_1 = \tau$ to satisfy the Hosios condition (Hosios (1990)), which ensures that the labor market matching is efficient.

For the parameters related wo the inter-firm cooperation contract, I set the prior for the separation probability of sourcing contract to a Beta distribution with mean 0.25, which implies that the average duration of sourcing contract is one year. The prior for the mean of the transaction cost distribution is Normal distributed with mean of 0 and a standard deviation of 0.2.