Emission Tax or Standard: The Roles of Productivity Dispersion and Abatement*

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Abstract

When a society wants to control aggregate emission under a certain target level, is it more desirable to impose a tax or a regulatory standard on emission? To answer this question, we explore a model where plants are heterogeneous in productivity and monopolistically competitive in the production of a set of varieties of (dirty-) goods whose by-product is emission. We establish three main results. First, if the technology on emission abatement is not available, then an emission tax unambiguously generates higher welfare than an emission standard. Second, if the plants can use the abatement technology and if there is no dispersion in productivity, then a standard induces higher output of the dirty goods than the tax. In this case, a standard is better than a tax if the monopoly power in the dirty-goods sector is sufficiently strong. Third, if the abatement technology and productivity dispersion are both present, a tax is better than a standard if either productivity dispersion is sufficiently wide or the monopoly power in the dirty-goods sector is weak. These results illustrate the importance of productivity dispersion in designing environmental policies. They contribute to the debate on whether a non-market instrument such as the regulatory standard can be better than a market-based instrument, such as a tax, for achieving an emission target.

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1 Introduction

The economics of climate change has attracted increasing attention in academics and public debate, especially with the publication of the Stern (2006) Review. The debate has focused on what level of CO_2-equivalent in the atmosphere that the world should set to achieve in the next five decades. There is wide disagreement on the scientific evidence on the pace of the accumulation of CO_2-equivalent in the atmosphere, the consequences of this accumulation on climate, and the effects of climate change on the world economy. In this paper, we address a different question: To achieve any given emission target, should an emission tax or a non-tradable emission standard be used? The predominant view on this question is that market-based instruments, such as emission taxes and tradable emission quotas, are more efficient than regulatory standards that are nontradable. We show that the answer is far from clear. In particular, we demonstrate that productivity dispersion in the economy and its interaction with abatement choices are important factors determining whether a market-based instrument or a nontradable regulatory standard is more desirable for achieving an emission target.

The choice between regulatory standards and market-based instruments is clearly important for environmental policy. A regulatory standard specifies the actions that a firm or individual must undertake to achieve environmental objectives. Firms and individuals cannot meet such a standard by just trading with others in the market. In contrast, instruments like emission taxes and tradable emission quotas work though the price system. Traditionally, regulatory standards had been the most common environmental policies. Starting from the 1970s, however, opinions have shifted to favor market-based instruments. For example, the Stern Review (part IV, p310) states that "a common price signal is needed across countries and sectors to ensure that emission reductions are delivered in the most cost-effective way...... [Both] taxes and tradable quotas have the potential to deliver emission reductions efficiently." With this assessment, the main debate on policy instruments has moved onto which market instruments should be used and, in particular, onto the choice between emission taxes and tradable emission quotas.

Our view is that this shift in the focus of the debate is premature on the ground of economics.[1] In order to assess the relative merit of market-based and regulatory standards,

[1] Freeman and Kolstad (2007) documents the past twenty years of experience in using market-based

we need to have models to emphasize productivity dispersion among firms and the interaction of this dispersion with firms' choices of emission abatement. It is evident that some heterogeneity among firms is necessary for making a relevant distinction between the two types of policies. In the simplest model where all firms are homogeneous and markets are perfectly competitive, the two types of policies must be equivalent: Since there is no net trade of quotas at the equilibrium price, the same level of emission can be achieved with the same efficiency by non-tradable quotas or regulatory standards.

Among many potential types of heterogeneity among firms, we emphasize production dispersion for two reasons. First, productivity dispersion has direct implications on how environmental policies affect an economy's efficiency in production. Second, productivity dispersion has been shown to be important for accounting for trade flows and for explaining how trade policies affect firms' trade decisions (e.g. Eaton and Kortum, 2002, and Melitz, 2003). It is natural to expect that productivity dispersion can play a similarly important role in determining how firms' abatement choices respond to environmental policies. Not much is known about this role in the literature.

Our model introduces emission and environmental policy into the Eaton-Kortum-Melitz model. There are two types of goods: a clean good and a dirty-goods composite. The composite aggregates a continuum of varieties that are not perfect substitutes. Each dirty-good variety is produced by one plant, and the sector of dirty goods is monopolistically competitive. The plants are heterogeneous in productivity. The production of a dirty-good variety generates emission as a by-product, and aggregate emission reduces the households' utility. A plant's emission increases with its input and, hence, with its output. However, a more productive plant has a lower emission intensity in the sense that its emission-output ratio is lower. We consider two environmental policies. One is an emission-intensity standard that requires a plant's emission-output ratio not to exceed a given level.[2] The other is an emission tax that a plant is required to pay for each unit of emission. As a response to the policies, a plant can put resources into emission abatement.

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instruments, in comparison with command-and-control policies such as regulatory standards.

[2] The regulatory standard examined here is a performance standard instead of a technology standard (see IPCC, 2007). A technology standard mandates specific pollution abatement technologies or production methods, such as specific CO_2 capture and storage methods on a power plant. A performance standard mandates specific environmental outcomes per unit of product such as a certain number of grams of CO_2 per kilowatt-hour of electricity generated.

In this model, the comparison between the tax and the standard is equivalent to the comparison between tradable emission permits and a nontradable instrument, because the price of a tradable permit will be equal to the tax rate. Given any arbitrary target on total emission, we evaluate the tax and the standard in three steps. First, we isolate the role of productivity dispersion by assuming that the abatement technology is not available. Second, we introduce the abatement technology but eliminate productivity dispersion. Finally, we put productivity dispersion back into the economy with the abatement technology to examine the interaction between the two elements.

When the plants have no access to the abatement technology, the emission tax induces a higher quantity of the dirty-goods composite and more varieties of the dirty goods than the standard; at the same time, the two policies yield the same quantity of the clean good. Thus, the tax yields higher welfare than the standard. This unambiguous ranking occurs because the standard forces some plants to shut down while the tax does not. Because the marginal productivity of the input in the dirty-goods composite is diminishing, shutting down some plants and moving the input to the remaining plants reduces average productivity and, hence, reduces the dirty-goods composite.

The abatement technology enables all plants to meet the requirement of the standard and remain operative. This eliminates one of the advantages of the tax and changes the comparison between the two policies significantly. In particular, when there is no productivity dispersion among the plants, the standard induces a higher quantity of the dirty-goods composite, and a lower quantity of the clean good, than the tax does. When the elasticity of substitution between different varieties of the dirty goods is sufficiently low, the quantity of the dirty-goods composite is sufficiently higher under the standard than under the tax, in which case the standard yields higher welfare than the tax.

Productivity dispersion interacts with the abatement choice. First, under the standard, the plants with relatively low productivity must spend more in abatement as a proportion to their input in production. This non-uniform abatement shifts the distribution of relative prices between different varieties of the dirty goods toward high levels (charged by low-productivity plants), which increases the price index of the dirty-goods composite and reduces the quantity of this composite. The tax does not change the distribution of relative prices of the varieties because it induces all plants to spend the same proportion of their input in abatement. Second, with productivity dispersion among the plants, the tax induces higher average productivity in the dirty-goods sector than the standard as in the case without the abatement technology. Both effects increase the advantage of the tax relative to the standard. Thus, when productivity is sufficiently dispersed among the plants, the tax yields a higher welfare than the standard.

One paper is related to the recent trade literature that emphasizes productivity dispersion among plants, e.g., Eaton and Kortum (2002) and Melitz (2003). Clearly, the environmental issues are different from trade issues. To some extent, the evaluation of the two environmental policies in our paper is also related to the earlier trade literature on tariffs versus quotas (e.g., Young and Anderson, 1980). However, our work has the following differences from this literature. First, productivity dispersion is an important element in our model but not in the earlier trade literature. Second, emission generates a negative externality to the society which does not have an apparent counterpart in the earlier trade literature. Third, emission is a by-product of a firm's production process. An emission policy is directly imposed on this by-product rather than the regular goods that a tariff or quota is imposed on. Fourth, the producers can use the abatement technology to reduce emission and different policies affect the abatement choice differently. With all these differences, it can be misleading to compare our results with those in the literature on tariffs versus quotas.

On the relative merit of price and quantity controls, there is a large literature originated from the seminal paper of Weitzman (1974).[3] For the examples of the applications to environmental issues, see Pizer (2002), Hepburn (2006) and Mandell (2008). A main message of this literature is that the relative merit of a price to a quantity instrument depends on whether the marginal-cost curve of abatement or the marginal-benefit curve of reduced emission is steeper and exposed to higher uncertainty. If the marginal-cost curve is steeper than the marginal-benefit curve and is exposed to higher uncertainty, then it is more beneficial to use a price instrument (such as an emission tax) which allows the aggregate quantity of emission to vary to reflect the large shift in the marginal cost. On the other hand, if the marginal-benefit curve is steeper and exposed to higher uncertainty,

[3] We do not survey this literature here. For some examples, see Laffont (1977) for incorporating subjective uncertainty, Yohe (1978) for examining additional sources of uncertainty and informational difficulty within a regulated heirarchy, and Kelly (2005) for a general-equilibrium framework.

then it is more beneficial to control the quantity and allow the price to vary to reflect the large shift in the benefit curve.

The main differences between our paper and this literature are as follows. First, we abstract from uncertainty entirely and uncover that productivity dispersion and its interaction with abatement choices are important elements for evaluating environmental policies. Second, we ask a different question. With uncertainty as the centerpiece, the literature on price versus quantity essentially asks the question whether it is more desirable to allow the price or the quantity of emission to vary more. In contrast, we fix the total quantity of emission at any arbitrary target and ask whether an emission tax or standard is better in welfare for achieving this target. Third, for the aforementioned literature to be relevant in the presence of heterogeneity among producers, both the quantity and the price instrument need to be tradable in the market. In contrast, we evaluate a market-based instrument (the tax) against a nontradable instrument (the standard).

In addition to uncertainty, we abstract from many other factors that may also be important for evaluating the relative merits of an emission tax and a standard. Examples include emission monitoring when producers have private information (see Montero, 2005) and practical difficulties in implementing an environmental policy (see Stern, 2006). Such abstraction enables us to clearly illustrate why productivity dispersion among producers is important for environmental policy and how it interacts with abatement choices.

2 The Model Environment

Consider a one-period economy that is populated by a unit measure of households. Each household is endowed with one unit of resource that can be supplied as the input in productive plants and receives dividends from a diversified portfolio of the plants. A household's utility function is $u(c, Q, M)$, where c is consumption of a clean good, Q consumption of a composite of the dirty-goods varieties, and M the aggregate level of emission per household. The composite of the dirty goods has the following form:

$$Q = \left[\int_{i \in I} (q_i)^{\frac{\varepsilon - 1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \quad \varepsilon > 1, \tag{1}$$

where q_i is the amount of consumption of variety i, the set I contains all the varieties of the dirty goods produced in the period, and ε is the elasticity of substitution between the varieties. The assumption $\varepsilon > 1$ is standard in the literature. Define the marginal rate of substitution between the dirty-goods composite and the clean good as

$$R(c, Q, M) = \frac{u_2(c, Q, M)}{u_1(c, Q, M)},$$

where the subscripts of u indicate partial derivatives.

Assumption 1 *(i)* $u_1 > 0$, $u_2 > 0$, $u_{11} < 0$ *and* $u_{22} < 0$; *(ii)* $u_3 < 0$; *(iii)* $R_3(c, Q, M) \leq 0$.

Part (i) of the assumption is standard. Part (ii) says that emission generates a negative externality on consumers. Part (iii) says that emission increases a household's desire for the clean good relative to the dirty goods.

The clean good is homogeneous and its production does not generate emission. The technology for producing the clean good is $w\ l_c$, where l_c is the input and $w > 0$ is a constant. For the sake of simplicity, we lump all types of inputs into one so that the production function is linear in this input. There is perfect competition in the clean-good sector, and so the wage rate in terms of the clean good is equal to the constant w. Throughout this paper, we will use the clean good as the numeraire.[4]

The dirty goods have varieties in a continuum, $[0, 1]$. Each variety is produced by at most one plant, and the dirty-goods sector is monopolistically competitive. At the beginning of the period, all plants are identical. A plant can choose at most one variety to produce. After the choice, the plant draws a productivity level x from the distribution (cdf) $G(.)$, with a support $[\underline{x}, \infty)$. We will refer to a plant with productivity x as plant x. Note that different plants that draw the same x produce different varieties. After the realization of x, a plant can choose whether or not to operate. Thus, depending on the environmental policy in place, the set of dirty goods produced in the economy may or may not be the same as the interval $[0, 1]$. If plant x operates, output is

$$q(x) = e^x l(x),$$

where $l(x)$ is the plant's input. Because the plant is the only one that produces the variety, it does not take the price of the variety as given; instead, it takes as given the demand curve for the variety.

[4]Including the clean good in the model is convenient because we can model all fixed costs and taxes in terms of the clean good, thereby simplifying accounting in the model.

Production of a dirty good generates emission. In the baseline model, the amount of emission of plant x is:

$$m(x) = b\ l(x), \quad b > 0. \tag{2}$$

A high-x plant generates more emission since it produces more output, but its emission-output ratio, $m(x)/q(x)$, is lower.[5] In an extended model, we will introduce an abatement technology which a plant can use to reduce emission. By choosing a level of abatement, $a(x)$, a plant can change its emission level to

$$m(x) = b\ l(x) \left(1 + \frac{a(x)}{l(x)}\right)^{-\frac{1}{\gamma}}, \quad \gamma > 0. \tag{3}$$

Let us express the level of abatement, a, in terms of the input so that the cost of the abatement is wa. The effectiveness of the abatement technology can be measured by $1/\gamma$. When $\gamma \to 0$, even a small amount of abatement can eliminate the plant's emission; when $\gamma \to \infty$, abatement does not reduce emission.

We evaluate two environmental policies: (i) an emission tax τ that requires a plant to pay τ (in terms of the clean good) for every unit of emission, and (ii) an emission standard s that requires a plant's emission-output ratio not to exceed s. The revenue from the tax is rebated to the households through lump-sum transfers.[6] Note that the emission standard is on a plant's emission-output ratio, rather than on the plant's level of emission. This specification makes sense when the plants are heterogeneous in productivity. If an emission standard requires a plant's emission not to exceed a certain level, instead, then a more productive plant will be more constrained by the standard than a less productive plant is. Similarly, the presence of heterogeneous productivity is the reason why we have assumed that a plant's emission-output ratio is a decreasing function of x. If a plant's emission-output ratio is an increasing function of the plant's productivity, instead, then an emission standard puts an upper bound on the productivity level below which a plant can operate. Since an emission standard in this case prevents more productive plants from operating, it does not seem to be a good policy.

[5] Although our main results can continue to hold with a general functional form of $m(x)$ that preserves this property, the simple form above makes the analysis tractable. For example, one can consider the specification $m(x) = m_0 q(x) + b\,[q(x)]^{\psi}$, where $m_0 > 0$, $b > 0$ and $\psi \in (0, 1)$.

[6] This assumption eliminates the need for government revenue as a potential difference between the tax and the standard. See Stern (2006, Part IV) for more discussions on this difference.

Another commonly debated policy is one that requires a plant to obtain an emission permit for each unit of emission. We do not examine this policy separately here because it is equivalent to the tax, as stated below:

Remark 2 *When an emission permit is tradable in a competitive market, it is equivalent to an emission tax, with the price of the permit being equal to the tax rate.*

Moreover, an emission standard can be interpreted as an emission permit that is granted free to the plants whose emission-output ratio does not exceed the constant s. In this sense, a main difference between an emission tax and an emission standard is that an emission tax directly affects a plant's marginal cost of production while an emission standard does not, provided the plant meets the standard.[7]

3 Equilibrium and Policies without Abatement

In this section, we characterize the equilibrium and evaluate the two policies when the abatement technology does not exist, i.e., when the emission process obeys (2). Section 4 will incorporate the abatement technology. To unify the notation for the two policies, we characterize the equilibrium when both policies are present.

3.1 Household's decisions

A household chooses consumption of the clean good, c, and consumption of the dirty goods, $(q_i)_{i \in I}$, to maximize utility. It is convenient to index a dirty good by the plant's productivity level, x, rather than the index i. For each plant with a particular x, let $l(x)$ be its input, $q(x)$ its output, $p(x)$ the price of its product and $m(x)$ its emission. Because each plant's productivity is drawn randomly according to the distribution G, the density of plants with any particular x is $G'(x)$.[8] Let X be the set of productivity levels of operative

[7] Another policy is to issue a production permit (license) that is necessary for a plant to produce dirty goods and that is sold in the market at a competitive price. In contrast to an emission tax or standard, such production permits do not directly restrict a plant's emission, because a plant that obtains a production permit can produce as much as desirable. For this reason, a production permit cannot affect a plant's choice of the abatement technology.

[8] It is important to bear in mind that even if two plants draw the same x, they necessarily produce different varieties. The quantity $q(x)$ is not the output of all plants that draw a particular x, but rather the output of each of these plants. The same clarification applies to $l(x)$ and $m(x)$.

plants. The composite of the dirty goods in (1) can be expressed as

$$Q = \left[\int_{x \in X} [q(x)]^{\frac{\varepsilon-1}{\varepsilon}} dG(x) \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{4}$$

The household chooses c and $\{q(x)\}_{x \in X}$ to maximize $u(c, Q, M)$ subject to (4) and the following budget constraint:

$$c + \int_{x \in X} p(x) q(x) dG(x) \leq w + \int_{x \in X} \pi(x) dG(x) + T.$$

Here, $\pi(x)$ is the dividend from plant x and T the lump-sum transfer from the government. This maximization problem yields the following optimality conditions:

$$p(x) = P \left(\frac{q(x)}{Q} \right)^{-1/\varepsilon}, \tag{5}$$

$$\frac{u_2(c, Q, M)}{u_1(c, Q, M)} = P, \tag{6}$$

where P is the price of the composite Q:

$$P = \left[\int [p(x)]^{1-\varepsilon} dG(x) \right]^{\frac{1}{1-\varepsilon}}. \tag{7}$$

3.2 Plants' Decisions

Consider a plant x in the dirty-goods sector that chooses to produce. The plant's profit is:

$$\pi(x) = p(x) q(x) - w l(x) - \tau m(x).$$

The plant faces the demand curve for its product, given by (5). It chooses input, $l(x)$, to maximize profit, taking as given the industry demand Q and the price index P. Substituting $p(x)$ from (5), $q(x)$ from the production function, and $m(x)$ from (2), we can express the plant's profit as

$$\pi(x) = P Q^{\frac{1}{\varepsilon}} \left[e^x l(x) \right]^{\frac{\varepsilon-1}{\varepsilon}} - (w + \tau b) l(x).$$

It is easy to verify that the plant's optimal input is

$$l(x) = Q \; e^{(\varepsilon-1)x} \left(\frac{(\varepsilon - 1) P}{\varepsilon (w + \tau b)} \right)^{\varepsilon}. \tag{8}$$

The plant's output is $q(x) = e^x l(x)$ and its emission is $m(x) = bl(x)$. We can calculate the price of the plant's product from (5) as

$$p(x) = \frac{\varepsilon}{\varepsilon - 1}(w + \tau b)e^{-x}. \tag{9}$$

Note that this price is a constant markup $(\frac{1}{\varepsilon-1})$ of the plant's effective marginal cost, $(w + \tau b)e^{-x}$. Moreover, the plant's maximized profit is:

$$\pi(x) = \frac{P^{\varepsilon}Q}{\varepsilon} e^{(\varepsilon-1)x} \left(\frac{\varepsilon - 1}{\varepsilon(w + \tau b)} \right)^{\varepsilon-1}. \tag{10}$$

Denote $x_0 \geq \underline{x}$ as the threshold level of productivity below which a plant chooses not to operate. The set of productivity levels observed in the economy is $X = [x_0, \infty)$. Since $\pi(x) > 0$ for all x, all plants choose to operate under the emission tax; that is, $x_0 = \underline{x}$. Under the emission standard, a plant x can operate only if $s \geq m(x)/q(x) = be^{-x}$. That is, $x_0 = \ln(b/s)$ under the emission standard. We summarize these two cases as

$$x_0 = \begin{cases} \ln(b/s), & \text{with the emission standard} \\ \underline{x}, & \text{without the tax.} \end{cases} \tag{11}$$

3.3 Aggregation and equilibrium

Let us denote total input in the production of dirty goods as $L = \int_{x_0}^{\infty} l(x)dG(x)$ and the average productivity in the dirty-goods sector as $\phi = Q/L$. Substituting $l(x)$ from (8), we obtain an expression for L. Similarly, substituting $q(x) = e^x l(x)$ into (4), we can obtain an expression for Q. Dividing these expressions for Q and L reveals that the average productivity in the dirty-goods sector is $\phi = \phi(x_0)$ where

$$\phi(x_0) = \left[\int_{x_0}^{\infty} e^{(\varepsilon-1)x} dG(x) \right]^{\frac{1}{\varepsilon-1}}. \tag{12}$$

The average of the plants' effective marginal costs of production is $(w+\tau b)/\phi$. Substituting (9) into (7) reveals that the price level of the dirty-goods composite is a constant markup over this average effective cost. That is, $P = P(\tau, \phi)$ where

$$P(\tau, \phi) \equiv \frac{\varepsilon}{\varepsilon - 1}(\frac{w + \tau b}{\phi}). \tag{13}$$

Let us refer to $(\tau, s, \bar{M}, T)$ as the policies, where $\bar{M}$ is the target level of aggregate emission. An equilibrium under the policies $(\tau, s, \bar{M}, T)$ consists of the set $X = [x_0, \infty)$, the functions $(l(x), q(x), p(x))_{x\in X}$, the aggregate levels (c, Q, P, L, M) that satisfy the following requirements: (i) Given the functions $(p(x))_{x\in X}$, a household's demand for the clean good, c, and the demand for each dirty good, $q(x)$, satisfy (5) and (6); (ii) Given (P, Q) and the demand function (5), a plant operates if and only if $x \geq x_0$, where x_0 satisfies (11), and if a plant operates, its choices of input and output satisfy (8) and $q(x) = e^x l(x)$; (iii) The levels of (P, L, M) satisfy (13), $L = Q/\phi$ and $M = bL$, where ϕ is given by (12); (iv) The resource market clears, i.e., $l_c = 1 - L$; (v) The market of the clean good clears, i.e., $c = wl_c$, and the markets of the dirty goods clear; (vi) The policy τ or s ensures aggregate emission not to exceed the target $\bar{M}$, i.e., $M \leq \bar{M}$, and the transfer T satisfies $T = \tau M$ under the tax and $T = 0$ under the standard.

Suppose that the emission target $\bar{M}$ is binding; that is, the economy produces $M > \bar{M}$ if there is no policy (see a precise condition for a binding target below). In this case, $M = \bar{M}$ in the equilibrium. We can determine an equilibrium as follows. Part (iii) above gives $L = \bar{M}/b$, $Q = \phi\bar{M}/b$ and $P = P(\tau, \phi)$, while parts (iv) and (v) give $c = w(1 - \frac{\bar{M}}{b})$. With these results, (6) required by part (i) becomes

$$R\left(w - w\frac{\bar{M}}{b}, \phi\frac{\bar{M}}{b}, \bar{M}\right) = P(\tau, \phi), \tag{14}$$

where $\phi = \phi(x_0)$. Equation (14) determines the level of the tax or the standard that is needed to implement the emission target $\bar{M}$. Note that the emission standard enters the above equation through $\phi(x_0)$, because x_0 is a function of s. After determining the required policy level, we can recover other equilibrium variables according to parts (i) through (v) of the above definition.

Let us denote the tax that achieves the emission target $\bar{M}$ as $\tau(\bar{M})$, and the standard as $s(\bar{M})$. Define $M_{\max}$ as level of $\bar{M}$ such that

$$R\left(w - w\frac{M_{\max}}{b}, \phi(\underline{x})\frac{M_{\max}}{b}, M_{\max}\right) = P(0, \phi(\underline{x})). \tag{15}$$

Lemma 3 *Suppose that the abatement technology does not exist. Assume that $[QR(c, Q, M)]$ is a strictly increasing function of Q for any given (c, M) and that $\lim_{Q\to 0}(QR) = 0$. The emission target $\bar{M}$ is binding if and only if $\bar{M} < M_{\max}$. Given any binding target $\bar{M}$, there is a unique equilibrium under each policy. Moreover, $\tau'(\bar{M}) < 0$ and $s'(\bar{M}) > 0$.*

The proof of this lemma appears in Appendix A. In addition to existence and uniqueness of an equilibrium, the lemma states the intuitive feature that the tax can be lower and the standard can be looser when the required emission target is higher.

3.4 Comparison between the two policies

Let us add the subscript τ to (x_0, L, P, Q, ϕ, u) under the tax and the subscript s to the variables under the standard. The following proposition compares the equilibrium under the tax with the equilibrium under the standard (see Appendix A for a proof):

Proposition 4 *Assume that the abatement technology does not exist. Given any binding emission target $\bar{M}$, the following results hold: (i) $L_\tau = L_s$ and $c_\tau = c_s$; (ii) $x_{0\tau} < x_{0s}$, $Q_\tau > Q_s$, $\phi_\tau > \phi_s$ and $P_\tau < P_s$; (iii) $u_\tau > u_s$.*

Result (i) is not surprising. Since a plant's emission is proportional to the plant's input, total emission is proportional to total input in the dirty-goods sector. Given the same emission target, total input in the dirty-goods sector must be the same under the two policies. As a result, total input in the clean-good sector must also be same under the two policies, and so consumption of the clean-good must be the same. Result (ii) states that, relative to the standard, the emission tax induces a larger set of varieties of the dirty goods to be produced, a higher level of consumption of the dirty-goods composite, a higher average level of productivity and a lower price of the dirty-goods composite. We will explain this result below. Since the tax generates higher consumption of the dirty-goods composite than the standard and the same level of consumption of the clean good, the tax dominates the standard in welfare, as stated in Result (iii).

The outcome $x_{0\tau} < x_{0s}$ in Result (ii) arises from the difference in how the two policies affect a plant's marginal cost and emission. The emission tax increases every plant's effective marginal cost of production, thereby reducing every plant's input, output and emission. However, since each plant charges a price that is a constant markup over the effective marginal cost, the increase in the marginal cost does not eliminate the plant's profit. Instead, every plant continues to operate with positive profit under the tax. In contrast, the emission standard does not affect a plant's effective marginal cost and emission. To meet the emission target at the aggregate level, the standard must induce some plants to shut down, i.e., the plants with low levels of x. As a result, fewer varieties are produced under the standard than under the tax.

The two policies have different effects on the average productivity. The tax does not change the average productivity, which is clear from (12). That is, the tax reduces total input in the dirty-goods sector and output of the dirty-goods composite in the same proportion. In contrast, the standard shuts down low-x plants. Because total input in the dirty-goods sector is the same under the two policies (for any given emission target), the standard shifts the input from low-x plants to high-x plants. A priori, one would suggest that this shift should increase the average productivity. Surprisingly, the opposite is true, as (12) clearly shows that the average productivity is a decreasing function of x_0.

How can it be the case that shutting down low-x plants and moving the input from these plants to high-x plants reduces the average productivity? To answer this question, it is important to recognize that different varieties of the dirty goods are not perfect substitutes in the composite, Q. Measured in terms of the contribution to Q, the input in a low-x plant does not have lower marginal productivity than the input in a high-x plant. Rather, the marginal productivity of the input in terms of the contribution to the composite must be the same for all x, because the resource used as the input in production is perfectly mobile across the plants. To verify this statement, note that the sum of the input in all plants with x is $G'(x)l(x)$. The marginal contribution of this input to Q is

$$\frac{\partial Q}{G'(x)\partial[l(x)]} = e^x \left(\frac{q(x)}{Q}\right)^{-1/\varepsilon} = \frac{\varepsilon}{\varepsilon - 1}\frac{w + \tau b}{P} = \phi(x_0).$$

The first equality follows from (4), the second from (5) and (9), and the last from (13). This derivation reveals how the mobility of the production factor ensures that the marginal contribution of the input to the composite be the same for all x. Although a low-x variety has a lower productivity in terms of x than a high-x variety, a smaller amount of a low-x variety is produced than a high-x variety. Since consumers value all varieties and the marginal utility of a variety is diminishing, a low-x variety generates a higher marginal utility (and hence a higher price) than a high-x variety. More precisely, in terms of the contribution to Q, productivity e^x is weighted by $[q(x)/Q]^{-1/\varepsilon}$. A low-$x$ variety has a lower relative quantity $q(x)/Q$ in the composite and hence a larger weight than a high-x variety. Weighted by this marginal utility, productivity is the same in all varieties.

With the above result on the value-weighted productivity of the input, we can obtain the average productivity equivalently as follows:

$$\frac{1}{L}\int_{x_0}^{\infty}\frac{\partial Q}{G'(x)\partial l(x)}[l(x)G'(x)]dx = \phi(x_0).$$

When a binding emission standard is imposed, low-x plants shut down and the productive resource is re-allocated from these plants to high-x plants. In the equilibrium, the marginal contribution of the input to the composite Q is equalized at a new level across the plants that remain operative. This new level must be lower than the level before the standard is imposed, because the shift of the productive resource is from low-output varieties to high-output varieties and because the marginal contribution of each variety to Q is diminishing. Put differently, the loss of varieties under the standard reduces the value-weight productivity of the input.

Now it is easy to understand the results $P_\tau < P_s$ and $Q_\tau > Q_s$. The price level is a constant markup of the average effective marginal cost, $(w + \tau b)/\phi$. Since average productivity is higher under the tax than under the standard, the average effective marginal cost is lower and, hence, the price level is lower under the tax. Moreover, higher productivity under the tax directly translates into higher output of the composite, because total input in the dirty-goods sector is the same under the two policies.

Note that productivity dispersion is necessary for a non-trivial comparison between the two policies in this model. If there is no dispersion in productivity across the plants, then all plants have to shut down under a binding standard while all plants continue to operate under the tax. In this case, the tax is evidently better than the standard. Introducing the abatement choice avoids this trivial comparison, as shown in the next section.

Finally, let us make a remark on the optimal policy. Because emission generates a negative externality on the households, there might be a trade-off between the emission target, $\bar{M}$, and consumption, Q. In this case, the optimal emission target is determinate. This optimal target may or may not be the same under the two policies. However, for whatever target that is optimal under the standard, Proposition 4 implies that there exists a tax rate that achieves the same target and higher welfare than the standard. Thus, welfare is higher under the tax than under the standard even when the emission target is set to the optimal level under each policy.

4 Equilibrium and Policy Analysis with Abatement

In this section, we characterize the equilibrium and evaluate the two environmental policies when the plants can use the abatement technology to control emission according to (3). With this analysis, we intend to accomplish two goals. First, we want to illustrate that the emission standard can sometimes achieve higher welfare than the tax by affecting the plants' abatement choice differently from the tax. Second, we want to uncover how the relative advantage of the two policies depends on several key features of the economy such as productivity dispersion, the elasticity of substitution between the varieties of the dirty goods and the effectiveness of abatement.

4.1 Equilibrium characterization under each policy

A household's decisions are the same as those in subsection 3.1. A plant's decisions need to be modified. Consider the tax first. With the input into production, $l(x)$, and the input in abatement, $a(x)$, profit of a plant x is

$$\pi(x) = PQ^{\frac{1}{\varepsilon}}[e^x l(x)]^{1-\frac{1}{\varepsilon}} - wl(x) - \tau b\left(1 + \frac{a(x)}{l(x)}\right)^{-\frac{1}{\gamma}} l(x) - wa(x),$$

where we have substituted the demand function for the plant's product, (5), and the emission process, (3). The plant chooses the abatement level, $a(x)$, and the input, $l(x)$, to maximize profit. It is easy to verify that the plant's optimal choices are

$$a(x) = l(x)\left[\left(\frac{\tau b}{\gamma w}\right)^{\frac{\gamma}{\gamma+1}} - 1\right], \tag{16}$$

$$l(x) = Q\ e^{(\varepsilon-1)x}\left[\frac{(\varepsilon-1)P}{\varepsilon k}\right]^{\varepsilon}, \tag{17}$$

where

$$k \equiv w(\gamma+1)\left(\frac{\tau b}{\gamma w}\right)^{\frac{\gamma}{\gamma+1}}. \tag{18}$$

By comparing (17) with its counterpart without the abatement choice, (8), we can interpret the constant k as the plant's marginal cost of production which incorporates the wage rate, the marginal cost of abatement and the tax on emission. The optimal choices above induce the following levels of output, price, and emission under the tax:

$$q(x) = Q\ e^{\varepsilon x}\left[\frac{(\varepsilon-1)P}{\varepsilon k}\right]^{\varepsilon}, \quad p(x) = \frac{\varepsilon k e^{-x}}{\varepsilon - 1}, \quad m(x) = b\left(\frac{\tau b}{\gamma w}\right)^{\frac{-1}{\gamma+1}} l(x). \tag{19}$$

As in the baseline model, the price of a plant's product is a constant markup over the plant's effective marginal cost of production, which is ke^{-x} in the current environment. Moreover, as in the baseline model, $\pi(x) > 0$ for all x, and so all plants operate under the tax. That is, the set of productivity levels observed in the economy is $X = [\underline{x}, \infty)$.

Under the emission standard, profit of a plant x is

$$\pi(x) = PQ^{\frac{1}{\varepsilon}}[e^x l(x)]^{1-\frac{1}{\varepsilon}} - wl(x) - wa(x).$$

The plant must meet the emission standard, i.e., $m(x)/q(x) \leq s$. With the emission process, (3), we can rewrite this requirement as $a(x) \geq l(x)\left[\left(\frac{b}{s}\right)^{\gamma} e^{-\gamma x} - 1\right]$. We allow the choice a to be negative as well as positive, provided $1 + \frac{a}{l} \geq 0$.[9] Solving the constrained maximization problem, we obtain the following optimal choices of a and l:

$$a(x) = l(x)\left[\left(\frac{b}{s}\right)^{\gamma} e^{-\gamma x} - 1\right], \tag{20}$$

$$l(x) = Q\ e^{[\varepsilon(\gamma+1)-1]x}\left[\frac{(\varepsilon-1)P}{\varepsilon w}\left(\frac{s}{b}\right)^{\gamma}\right]^{\varepsilon}. \tag{21}$$

The optimal choices above induce the following levels of output, price and emission under the standard:

$$q(x) = Q\ e^{\varepsilon(\gamma+1)x}\left[\frac{(\varepsilon-1)P}{\varepsilon w}\left(\frac{s}{b}\right)^{\gamma}\right]^{\varepsilon}, \tag{22}$$

$$p(x) = \frac{\varepsilon e^{-(\gamma+1)x}}{\varepsilon-1} w \left(\frac{b}{s}\right)^{\gamma}, \quad m(x) = sq(x). \tag{23}$$

These expressions reveal that a plant's marginal cost is $w\left(\frac{b}{s}\right)^{\gamma} e^{-\gamma x}$, which includes the wage rate and the marginal cost of abatement. The effective marginal cost is $w\left(\frac{b}{s}\right)^{\gamma} e^{-(\gamma+1)x}$.

In contrast to the model without abatement, the model with abatement implies that $\pi(x) > 0$ for all x and every plant meets the standard. All plants are able to operate under the standard, because a plant can spend enough in abatement and cut production sufficiently to meet the standard. Since all varieties are produced under both policies, we will omit the notation for the interval over which the integrals are computed in this section, which is $[\underline{x}, \infty)$.

[9] The interpretation of a choice $a < 0$ is that the plant uses a production technology that produces more emission than the production technology in the baseline model.

As in the baseline model, we define the average productivity in the dirty-goods sector as $\phi = Q/L$. We can compute the aggregate levels of the input, L, the composite of the dirty goods, Q, the abatement, A, the policy level required to achieve an emission target, τ or s, and consumption of the clean good, c. Define the equilibrium by incorporating abatement in the definition in the baseline model. The following lemma summarizes the aggregate results and determines the equilibrium (see Appendix B for a proof):

Lemma 5 *Assume that the abatement technology is available. Under either policy, there is a unique equilibrium, where $L = Q/\phi$ and Q is determined by (6). Under the emission tax, $\phi = \phi(\underline{x})$, where the function $\phi(.)$ is given by (12), and other aggregate quantities and prices are as follows:*

$$\tau = \frac{\gamma w}{b}\left(\frac{bL}{M}\right)^{\gamma+1}, \quad P = \frac{\varepsilon w(\gamma+1)}{(\varepsilon-1)\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}, \tag{24}$$

$$A = L\left[\left(\frac{bL}{M}\right)^{\gamma} - 1\right], \quad c = w - \frac{wQ}{\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}. \tag{25}$$

Under the emission standard, the aggregate quantities and prices are:

$$\phi = \left[\int e^{(\gamma+1)(\varepsilon-1)x} dG(x)\right]^{\frac{\varepsilon}{\varepsilon-1}} \Big/ \int e^{[\varepsilon(\gamma+1)-1]x} dG(x) \,, \tag{26}$$

$$s = \frac{M}{L}\frac{\int e^{[\varepsilon(\gamma+1)-1]x} dG(x)}{\int e^{\varepsilon(\gamma+1)x} dG(x)}, \quad P = \frac{\varepsilon w\lambda}{(\varepsilon-1)\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}, \tag{27}$$

$$A = L\left[\left(\frac{bL}{M}\right)^{\gamma}\lambda - 1\right], \quad c = w - \frac{wQ}{\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}\lambda, \tag{28}$$

where

$$\lambda = \frac{\left[\int e^{(\varepsilon-1)(\gamma+1)x} dG(x)\right]\left[\int e^{\varepsilon(\gamma+1)x} dG(x)\right]^{\gamma}}{\left[\int e^{[\varepsilon(\gamma+1)-1]x} dG(x)\right]^{\gamma+1}}. \tag{29}$$

4.2 The case with no dispersion in productivity

To illustrate the importance of productivity dispersion for policy evaluation, we consider first the case where there is no dispersion in productivity among the plants. Let x be the level of productivity of all plants in this case. Adding the subscript τ to the variables under the tax and the subscript s to the variables under the standard, we have the following proposition (see Appendix B for a proof):

Proposition 6 *When there is no productivity dispersion, the equilibrium with the abatement choice yields* $\phi_s = \phi_\tau$, $\lambda = 1$, $Q_s > Q_\tau$, $L_s > L_\tau$, $A_s > A_\tau$ *and* $c_s < c_\tau$ *for any given emission target* M. *Moreover, a sufficient condition for* $u_s > u_\tau$ *is* $\varepsilon \leq 1 + \frac{1}{\gamma}$. *On the other hand, if* ε *is sufficiently large, then* $u_s < u_\tau$.

This proposition shows that the abatement choice can sometimes reverse the results obtained without abatement. Recall that without abatement, the tax induces a higher quantity of the dirty-goods composite and higher utility than the standard does (see Proposition 4). Especially, in the limit case without productivity dispersion, a binding standard induces all plants to shut down while a tax keeps all plants operative. In the same limit case but with abatement, all plants are operative under both policies. With no productivity dispersion, the average productivity is clearly the same under both policies. The standard induces higher input in the dirty-goods sector, higher output of the dirty-goods composite and higher abatement. In addition, if the monopoly power is sufficiently strong in the sense that ε is small, then the standard generates higher welfare than the tax.

Why is the emission standard possibly better than the tax when abatement is available? One reason is that the abatement technology enables all plants to operate under the standard. The other is that, with the abatement choice and homogeneous productivity, the standard creates less upward pressure on the price level of the dirty-goods composite than the tax does. Under the standard, the additional cost to a plant is the abatement cost. Under the tax, in contrast, a plant's additional cost consists of both the abatement cost and the tax on emission. As a result, a plant's marginal cost of production is lower under the standard than under the tax for the same emission level and the same input. Since the price of a plant's product is a constant markup over the effective marginal cost of production, the price is also lower under the standard than under the tax. When there is no dispersion in productivity, this difference between the two policies' effects on individual plants' prices translates into the difference in the price level of the dirty-goods composite. The lower price level under the standard induces the households to consume more dirty goods and less clean good. As a result, more resource is employed in the production of the dirty goods under the standard than under the tax. With a higher input in production of the dirty goods, abatement is also higher under the standard in order for the sector to meet the requirement of the standard. This implies that consumption of the clean good is lower under the standard than under the tax, and so the welfare ranking between the two policies depends on the relative change in consumption of the two sectors' goods. When the monopoly power in the dirty-goods sector is sufficiently strong, the markup of price on the marginal cost is high, and a small difference in the marginal cost can translate into a large difference in price. In this case, the increase in the dirty-goods composite under the standard relative to the tax is sufficiently large to outweigh the decrease in the clean good, and so the standard induces higher utility than the tax.

To support the above explanation, recall that a plant's effective marginal cost of production is $w(\gamma+1)(\frac{\tau b}{\gamma w})^{\frac{\gamma}{\gamma+1}}e^{-x}$ under the tax and $w\left(\frac{b}{s}\right)^{\gamma}e^{-(\gamma+1)x}$ under the standard. When there is no dispersion in productivity, each plant's input, abatement, and emission are equal to their industry average. That is, $l(x) = L$, $a(x) = A$ and $m(x) = M$. From the equation $m(x) = M$, we can solve the policy level (s or τ) that is required to meet the target M. With this policy level, the effective marginal cost of production is $w(\gamma+1)(\frac{bL}{M})^{\gamma}e^{-x}$ under the tax and $w(\frac{bL}{M})^{\gamma}e^{-x}$ under the standard. Clearly, the latter is lower than the former by a factor $(\gamma+1)$. Since the price level is $\varepsilon/(\varepsilon-1)$ times the effective marginal cost, it is also lower under the standard than under the tax by a factor $(\gamma+1)$. In this case, (6) implies that the solution for Q is higher under the standard than under the tax.

4.3 The role of productivity dispersion

To understand the role of productivity dispersion in the evaluation of the two policies, let us assume that x is exponentially distributed:

$$G(x) = 1 - e^{-(x-\underline{x})/\delta}, \quad \delta \in (0, \bar{\delta}), \tag{30}$$

where $\bar{\delta} = \min\{\frac{1}{2}, \frac{1}{\varepsilon(\gamma+1)}\}$. This distribution implies that productivity $z = e^x$ is distributed according to the Pareto distribution: $G_z(z) = 1 - \left(\frac{e^{\underline{x}}}{z}\right)^{1/\delta}$. The restriction $\delta < 1/[\varepsilon(\gamma+1)]$ is required for λ to be finite, while the restriction $\delta < 1/2$ is required for the variance of e^x to be finite. Under the restriction $\delta < 1/2$, the mean of e^x is $\frac{e^{\underline{x}}}{1-\delta}$ and the variance is $\frac{\delta^2 e^{2\underline{x}}}{(1-2\delta)(1-\delta)^2}$. If we fix the mean of e^x at any arbitrary level $\bar{z} > 0$ by setting $\underline{x} = \ln[(1-\delta)\bar{z}]$, then the variance of e^x is $\frac{(\delta\bar{z})^2}{1-2\delta}$. Since this variance is increasing in δ, we refer to δ as the dispersion of productivity among the plants.

We first consider an economy with the following utility function:

$$u(c, Q, M) = U(c + v(M)Q), \text{ with } U' > 0,\ U'' < 0,\ v > 0,\ v' < 0. \tag{31}$$

With this utility function, the marginal rate of substitution between the dirty-goods composite and the clean good is $u_2/u_1 = v(M)$. For any given emission target M, the equilibrium price of the dirty-goods composite is a constant $P = v(M)$. This feature simplifies the analysis significantly. Later, we will illustrate that the qualitative results extend to a more general utility function.

With (31), we can solve equilibrium Q explicitly from (6) as

$$Q = \begin{cases} \frac{M}{b}\left[\frac{(\varepsilon-1)v(M)}{\varepsilon w}\frac{(\phi_\tau)^{\gamma+1}}{\gamma+1}\right]^{1/\gamma}, & \text{with the tax} \\ \frac{M}{b}\left[\frac{(\varepsilon-1)v(M)}{\varepsilon w}\frac{(\phi_s)^{\gamma+1}}{\lambda}\right]^{1/\gamma}, & \text{with the standard.} \end{cases} \tag{32}$$

For any given emission target, the equilibrium value of Q differs under the two policies in two aspects. One is the difference in the average productivity. The other is that there is a constant $(\gamma + 1)$ under the tax, while the corresponding constant is λ under the standard. The following proposition compares the two policies (see Appendix C for a proof):

Proposition 7 *Assume that the abatement technology is available and there is dispersion in productivity. It is always true that $\phi_s < \phi_\tau$. Define*

$$\varepsilon_0 = \frac{1}{\gamma}\left[(\gamma+1)^{(\gamma+1)/\gamma} - 1\right] \qquad (> 1).$$

With the utility function (31), $P_\tau = P_s = v(M)$, and the following results hold: (i) $\left(\frac{Q}{L+A}\right)_s < \left(\frac{Q}{L+A}\right)_\tau$; (ii) If $\varepsilon \geq \varepsilon_0$, then $u_s < u_\tau$ for all δ and, if $\varepsilon < \varepsilon_0$, then there exists $\delta_0 \in (0, \bar{\delta})$ such that $u_s > u_\tau$ iff $\delta < \delta_0$; (iii) $u_s > u_\tau \Longrightarrow Q_s > Q_\tau \Longrightarrow L_s > L_\tau \Longrightarrow A_s > A_\tau$, and $L_s > L_\tau \Longrightarrow c_s < c_\tau$; (iv) The emission target that maximizes utility is the same under the both policies and is given by $M^* = \arg\max M\,[v(M)]^{(1+\gamma)/\gamma}$.

Productivity dispersion changes the relative merit of the two policies significantly. First, the tax induces a higher composite of the dirty goods than the standard when productivity dispersion is sufficiently wide. Second, sufficiently wide dispersion of productivity can make the tax dominate the standard in welfare for all values of the monopoly power in the dirty-goods sector. These differences from Proposition 6 arise from the fact that productivity dispersion reduces the upward pressure that the tax generates on the price index of the dirty-goods composite relative to the standard. The reduction comes in two ways. First, with productivity dispersion, average productivity is higher under the tax than under the standard, which reduces price. Second, under the standard, abatment increases prices by more at high levels than at low levels. This change in the distribution of relative prices puts additional upward pressure on the price index of the dirty-goods composite, as reflected by the factor $\lambda > 1$ in (27). In contrast, under the tax, abatement increases prices by the same proportion in all plants, and so it does not change the distribution of relative prices. When productivity dispersion is sufficiently wide, these two effects together dominate the effect through an individual plant's marginal cost. In this case, the standard generates a stronger upward push on the price level than does the tax. Although the equilibrium price must be the same under the two policies when the utility function has the form (31), the additional upward pressure on the price level under the standard must be absorbed by a larger fall in output of the dirty-goods composite. Hence, utility can be lower under the standard than under the tax.

Let us first explain why the average productivity in the dirty-goods sector is lower under the standard than under the tax. Recall that this difference exists in the baseline model because the standard induces low-x plants to shut down. With the abatement choice, all varieties continue to be produced under either policy, but the reason for the productivity difference under the two policies is similar to that in the baseline model. That is, the standard shifts the input from low-x plants to high-x plants while the tax does not. To see why the shift occurs under the standard, note that in order to meet the emission standard, a low-x plant must spend more in abatement in proportion to its production. That is, the ratio of the input in abatement to production, $a(x)/l(x)$, is a decreasing function of x under the standard. As a result, a high-x plant employes more resources in production than a low-x plant, above and beyond what the difference in productivity alone calls for. Because the marginal contribution of each variety to the composite Q is diminishing, this shift of the input from low-output plants to high-output plants reduces the average contribution of the input to the composite, as measured by ϕ. This shift does not occur under the tax because the ratio of abatement to the input in production is constant across the plants under the tax. Thus, the average productivity is lower under the standard. Note that this result does not rely on the particular utility function (31).

The measure of productivity, ϕ, counts only the input in production of the dirty goods but not the input in abatement. Total input in the dirty-goods sector is $(L + A)$. Thus, a measure of the effective productivity in the dirty-goods sector is $Q/(L + A)$. Part (i) of Proposition 6 says that, when the utility function has the form in (31), the effective productivity is also lower under the standard than under the tax.[10]

Next, we explain how the abatement choice affects the distribution of relative prices between plants, thereby affecting the price level of the dirty-goods composite. Since the tax is proportional to a plant's emission, it increases all plants' marginal cost by the same proportion. All plants spend the same amount in abatement in proportion to their input in production. As a result, prices of all varieties increase by the same proportion under the tax, leaving the relative price between any two plants unchanged. In contrast, under the standard, low-x plants must spend more in abatement in proportion to production than high-x plants do in order to meet the requirement of the standard. Note that under monopolistic competition, low-x plants produce less and charge higher prices than high-x plants do. By increasing low-x plant's marginal costs by more than high-x plants', the abatement choice increases low-x plants' prices even more, thus tilting the distribution of relative prices toward high prices under the standard. As said earlier, this shift in the distribution of prices toward high prices adds upward pressure on the price level of the dirty-goods composite.

Now we explain why the tax can dominate the standard in welfare for all values of the monopoly power in the dirty-goods sector. When productivity dispersion is sufficiently wide, the standard induces much lower average productivity than the tax does and significantly shifts the distribution of relative prices toward high levels. The resulted upward pressure on the price level of the dirty-goods composite is sufficiently higher under the standard than under the tax, regardless of the value of ε. To restore the equilibrium, the quantity of the composite must fall by a sufficiently larger amount under the standard. This large fall in consumption of the composite reduces utility under the standard relative to the utility under the tax, as stated in part (ii) of Proposition 7.

Let us turn to the remaining parts of Proposition 7. Part (iii) is easy to understand.

[10] As in the baseline model, one can compute the effective marginal productivity of the input in each plant x as $\frac{1}{G'(x)}\partial Q/\partial[l(x) + a(x)]$ and verify that it is equal to $Q/(L + A)$.

First, for utility to be higher under the standard than under the tax, the standard must generate less pressure on the price index of the dirty-goods composite, in which case the composite decreases by less under the standard than under the tax. Second, since productivity is lower under the standard than under the tax, the input in the dirty-goods sector must be higher under the standard in order to produce a higher composite of the dirty goods than under the tax. Third, with (3), abatement is proportional to the input in production. Thus, a higher input in the production of the dirty-goods composite also calls for a higher level of abatement. Finally, when the input in the dirty-goods sector is higher under the standard, the input in the clean-good sector is lower, which contributes to lower consumption of the clean good under the standard than under the tax.

Part (iv) of Proposition 7 describes the optimal emission target, i.e., the target that maximizes the representative household's utility. With (31), the optimal level of c is proportional to vQ. Since vQ depends on M only through the term $Mv^{(1+\gamma)/\gamma}$, so does the utility level. The optimal target maximizes this term regardless of which policy is used to implement the target. This result implies that parts (i) - (iii) of Proposition 7 continue to hold when the emission target is set to the optimal level under each policy.

The other side of the story in Proposition 7 is that the standard can still be better than the tax if productivity dispersion is not wide and if the monopoly power in the dirty-goods sector is strong. This result extends Proposition 6 from the limit $\delta = 0$ to a neighborhood on the right side of $\delta = 0$. As a whole, Proposition 7 shows that in order to evaluate the relative merit of the two policies, one must have good information on how dispersed productivity is and how competitive the country's industry is.[11]

We conclude this section by illustrating that the results in Proposition 7 can be extended beyond the utility function in (31). Consider the following utility function:

$$u(c, Q, M) = \{\alpha c^\rho + (1-\alpha)[v(M)Q]^\rho\}^{1/\rho}, \ \alpha, \rho \in [0,1], \ v > 0, \text{ and } v' < 0.$$

The restriction $\rho \geq 0$ is imposed to satisfy Assumption 1. The case $\rho = 1$ corresponds to (31), where the clean good and the dirty-goods composite are perfect substitutes. The

[11] Note that the effectiveness of the abatement technology, $1/\gamma$, also plays a role since the critical levels ε_0 and δ_0 in Proposition 7 depend on γ. In the limit $\gamma \to 0$, even a tiny amount of input in abatement can reduce emission to zero. In this case, the two policies are equivalent. In the opposite limit $\gamma \to \infty$, the abatement is not effective at all, and the model approaches the baseline model where the tax induces higher welfare than the standard.

case $\rho = 0$ (Cobb-Douglas) is also analytically tractable and the results are the same as in Proposition 7 after a modification of ε_0. For general values of ρ, we cannot solve for Q analytically and so we solve it numerically. Since the example is only illustrative, we let $v(M) = M^{-\kappa}$ and fix some of the parameters as follows:

$$\alpha = 0.6,\ \gamma = 0.2,\ b = 3,\ w = 1,\ \underline{x} = \ln(1-\delta),\ \kappa = 0.5$$

The chosen value $\underline{x}$ implies that e^x has a mean 1 and a variance $\frac{\delta^2}{1-2\delta}$. We explore different values of $(\delta, \varepsilon, \rho)$. For each $\rho \in [0, 1]$, we find the region of (δ, ε) in which the standard yields higher welfare than the tax. For three values of ρ, 0.1, 0.5 and 0.8, Figure 1 depicts the curve in the (δ, ε) plane below which the standard dominates the tax. Thus, for all three values of ρ, the standard dominates the tax in welfare if and only if δ and ε are small. This result is consistent with Proposition 7. In addition, when the elasticity of substitution between the clean good and the dirty-goods composite decreases, i.e., when ρ increases, the curve moves slightly outward for middle values of δ, but inward for very small values of δ. That is, the less substitutable the clean good and dirty goods are, the more likely the standard dominates if there is some market power (and if δ is large enough).

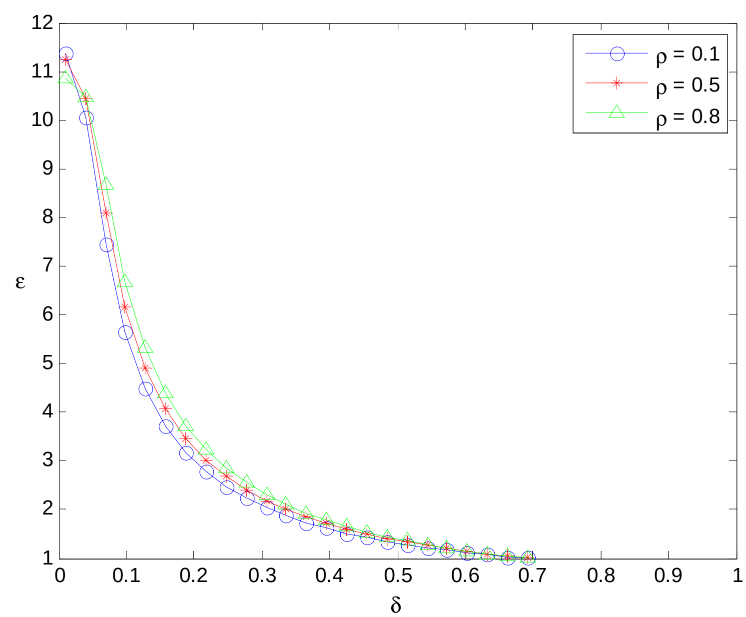

Figure 1. Regions of (δ, ε) in which the standard dominates the tax

5 Conclusion

When a society wants to control aggregate emission under a certain target level, is it more desirable to impose a tax or a regulatory standard on emission? To answer this question, we explore a model where plants are heterogeneous in productivity and monopolistically competitive in the production of a set of varieties of (dirty-) goods whose by-product is emission. We establish three main results. First, if the technology on emission abatement is not available, then an emission tax unambiguously generates higher welfare than an emission standard. Second, if the plants can use the abatement technology and if there is no dispersion in productivity, then a standard induces higher output of the dirty goods than the tax. In this case, a standard is better than a tax if the monopoly power in the dirty-goods sector is sufficiently strong. Third, if the abatement technology and productivity dispersion are both present, a tax is better than a standard if either productivity dispersion is sufficiently wide or the monopoly power in the dirty-goods sector is weak.

Our results illustrate the importance of productivity dispersion and its interaction with abatement choices in designing environmental policies. They demonstrate when and why a non-market instrument such as the regulatory standard can be better than a market-based instrument, such as an emission tax, for achieving an emission target. Both the focus on productivity dispersion and a fixed emission target deviate from the economics literature on the choice of environmental policy. The latter addresses the question whether it is more desirable to let the price or the aggregate quantity of emission to fluctuate when there is uncertainty in the marginal cost and benefit of emission reduction.

There are many directions in which one can explore further the importance of productivity dispersion for environmental policies. For example, with productivity dispersion, different policies can have different effects on plants' exit and entry. It is interesting to examine both the long-run effects of policies and the dynamics. Li and Sun (2009) calibrate such a dynamic model and quantitatively examine the effects of an emission tax and an emission standard.

Appendix

A Proofs of Lemma 3 and Proposition 4

We establish Lemma 3 first. Under the tax, (14) can be written as

$$R\left(w - w\frac{\bar{M}}{b}, \phi(\underline{x})\frac{\bar{M}}{b_0}, \bar{M}\right) = P(\tau, \phi(\underline{x})).$$

For any given $\bar{M}$, the left-hand side of the above equation is independent of τ and the right-hand side is an increasing function of τ. Thus, there exists a unique level of τ, denoted $\tau(\bar{M})$, that solves the above equation. With this level of the tax, other equilibrium variables are uniquely determined as in the main text. Moreover, the target $\bar{M}$ is binding if and only if $\tau(\bar{M}) > 0$. The latter requirement is equivalent to the condition that the left-hand side of the above equation is strictly greater than $P(0, \phi(\underline{x}))$. Since the left-hand side of the above equation is a strictly decreasing function of $\bar{M}$ (see Assumption 1), the target is binding if and only if $\bar{M} < M_{\max}$. It is easy to see that $\tau'(\bar{M}) < 0$ and $\tau(M_{\max}) = 0$.

Under the emission standard, $x_0 = \ln(b/s)$, and (14) can be written as

$$\phi\ R\left(w - w\frac{\bar{M}}{b}, \phi\frac{\bar{M}}{b}, \bar{M}\right) = \frac{\varepsilon}{\varepsilon - 1}w,$$

where $\phi = \phi(\ln(b/s))$. For any given $\bar{M}$, the assumption on qR imposed in Lemma 3 ensures that the left-hand side of the above equation is a strictly increasing function of ϕ and reaches 0 at $\phi = 0$. There is a unique level of ϕ, denoted $\phi_s(\bar{M})$, that solves the above equation. The implied standard can be calculated from $\phi_s(\bar{M}) = \phi(\ln(b/s(\bar{M})))$ and other equilibrium variables can be uniquely determined as in the main text. Moreover, the target $\bar{M}$ is binding iff $\ln(b/s(\bar{M})) > \underline{x}$, i.e., iff $\phi_s(\bar{M}) < \phi(\underline{x})$. This requirement is equivalent to $\bar{M} < M_{\max}$. Furthermore, it is easy to verify that $\phi_s'(\bar{M}) > 0$ and $\phi_s(M_{\max}) = \phi(\underline{x})$. Hence, $s'(\bar{M}) > 0$ and $s(M_{\max}) = be^{-\underline{x}}$. This completes the proof of Lemma 3.

Now we prove Proposition 4 by verifying statements (i)-(iii) in the proposition. Statement (i) is evident, since $L = \bar{M}/b$ and $c = w(1 - L)$ under both policies. For statement (ii), the proof of Lemma 3 has already established $x_{0\tau} = \underline{x} < \ln(b/s) = x_{0s}$ for any binding target. Because $\phi(x_0)$ defined by (12) is a strictly decreasing function of x_0, then $\phi_\tau > \phi_s$. Since $Q = \phi\bar{M}/b$, then $Q_\tau > Q_s$. Recall that $P = R(c, Q, M)$ and that $R(c, Q, M)$ is a strictly decreasing function of Q. We have $P_\tau = R(c, Q_\tau, \bar{M}) < R(c, Q_s, \bar{M}) = P_s$. Finally, $u_\tau = u(c, Q_\tau, \bar{M}) > u(c, Q_s, \bar{M}) = u_s$. QED

B Proofs of Lemma 5 and Proposition 6

For Lemma 5, we only derive the formulas for the tax and prove that there exists a unique equilibrium. The derivation and the proof for the standard are similar and, hence, are omitted. Under the tax, we substitute $q(x)$ from (19) into (1) to compute Q and aggregate $l(x)$ from (17) to compute L, which yields $\phi = Q/L = \phi(\underline{x})$, where $\phi(.)$ is the function defined by (12). Aggregating $m(x)$ in (19), we have $M = b(\frac{\tau b}{\gamma w})^{\frac{-1}{\gamma+1}} L$. Inverting this result yields τ as the function of (L, M) in (24). Substituting $p(x)$ from (19) into (7), we obtain $P = \frac{\varepsilon k}{(\varepsilon-1)\phi}$, where k is a function of τ given by (18). Substituting τ, we obtain

$$k = w(\gamma + 1)\left(\frac{bL}{M}\right)^{\gamma}, \tag{33}$$

and, hence, P is as in (24). Aggregating $a(x)$ in (16) and substituting τ from (24), we obtain A as in (25). Since total input in the dirty-goods sector is $(L + A)$, consumption of the clean good is $c = w(1 - L - A)$. Substituting A, we obtain c as in (25). The quantity Q is determined by (6). Proving that there exists a unique equilibrium amounts to proving that there is unique solution for Q. Substituting P from (24), we have

$$\frac{u_2(c(Q), Q, M)}{u_1(c(Q), Q, M)} = \frac{\varepsilon w(\gamma + 1)}{(\varepsilon - 1)\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}, \tag{34}$$

where $c(Q)$ is given by (25). Under Assumption 1, the left-hand side of (34) is a strictly decreasing function of Q, while the right-hand side is a strictly increasing function of Q. With these features, it is easy to prove that there is a unique solution for Q to (34). This completes the proof of Lemma 5.

For Proposition 6, we assume that the measure of plants is one without loss of generality. When all plants have the same productivity, $l(x) = L$, $a(x) = A$, $q(x) = Q$, and $m(x) = M$. It is evident that $\phi_s = \phi_\tau = e^x$ and $\lambda = 1$. Substituting the pricing formulas in (24) and (27) into (6), we obtain:

$$\frac{u_2(c, Q, M)}{u_1(c, Q, M)} = \frac{\sigma\varepsilon w}{(\varepsilon - 1)\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma},$$

where $\sigma = \gamma + 1$ under the tax and $\sigma = 1$ under the standard. Under both policies, $c = w - \frac{wQ}{\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}$ and $\phi = e^{x}$ in the above equation. Denote the solution to the above equation as $Q(\sigma)$. It is clear that $Q'(\sigma) < 0$. Thus, $Q_s > Q_\tau$. From (25), (28) and $L = Q/\phi$, it is clear that $L_s > L_\tau$, $A_s > A_\tau$, and $c_s < c_\tau$. Express the utility level as $\hat{u}(\sigma) \equiv u(c(\sigma), Q(\sigma), M)$. Then, $u_s > u_\tau$ iff $\hat{u}(1) > \hat{u}(\gamma + 1)$. It can be verified that $\hat{u}'(\sigma) < 0$ iff $\sigma > (\gamma+1)(\varepsilon-1)/\varepsilon$. If $1 \geq (\gamma+1)(\varepsilon-1)/\varepsilon$, i.e., if $\varepsilon \leq 1 + \frac{1}{\gamma}$, then $\hat{u}'(\sigma) < 0$ for all $\sigma > 1$. In this case, $u_\tau = \hat{u}(\gamma+1) < \hat{u}(1) = u_s$. On the other hand, if $\varepsilon \to \infty$, then $\hat{u}'(\sigma) > 0$ for all $\sigma < \gamma + 1$. In this case, $u_\tau = \hat{u}(\gamma+1) > \hat{u}(1) = u_s$. QED

C Proof of Proposition 7

Using (30), we compute the average productivity as

$$\phi = \begin{cases} [1-\delta(\varepsilon-1)]^{\frac{-1}{\varepsilon-1}} e^{\underline{x}}, & \text{with the tax} \\ \frac{1+\delta-\delta\varepsilon(\gamma+1)}{[1-\delta(\varepsilon-1)(\gamma+1)]^{\frac{\varepsilon}{\varepsilon-1}}} e^{\underline{x}}, & \text{with the standard.} \end{cases} \tag{35}$$

The statement $\phi_s < \phi_\tau$ is true if and only if

$$\frac{\varepsilon}{\varepsilon-1}\ln\left[1-\delta(\varepsilon-1)(\gamma+1)\right] - \ln\left[1+\delta-\delta\varepsilon(\gamma+1)\right] - \frac{1}{\varepsilon-1}\ln\left[1-\delta(\varepsilon-1)\right] > 0.$$

Temporarily denote the left-hand side as $LHS(\delta)$. Note that $LHS(0) = 0$ and $LHS(\frac{1}{\varepsilon(\gamma+1)}) > 0$. Also, $LHS'(\delta)$ has the same sign as the following expression:

$$\frac{\delta\varepsilon\gamma(\gamma+1)}{\left[1-\delta(\varepsilon-1)(\gamma+1)\right]\left[1+\delta-\delta\varepsilon(\gamma+1)\right]} + \frac{1}{\left[1-\delta(\varepsilon-1)\right]}.$$

Thus, $LHS'(\delta) > 0$. For all $\delta > 0$, $LHS(\delta) > LHS(0) = 0$.

With the utility function (31), it is clear that $P_s = P_\tau = v(M)$. For part (i) of the proposition, we can compute

$$\frac{Q}{L+A} = \begin{cases} \frac{\varepsilon w}{(\varepsilon-1)v(M)}(\gamma+1), & \text{with the tax} \\ \frac{\varepsilon w}{(\varepsilon-1)v(M)}, & \text{with the standard.} \end{cases}$$

It is evident that $Q/(L+A)$ is higher under the tax than under the standard.

For other parts of the proposition, we substitute G from (30) into (29) to compute

$$\lambda = \frac{\left[1+\delta-\delta\varepsilon(\gamma+1)\right]^{\gamma+1}}{\left[1-\delta\varepsilon(\gamma+1)\right]^{\gamma}\left[1-\delta(\varepsilon-1)(\gamma+1)\right]}\ (>1). \tag{36}$$

For part (ii), we use (32) to solve for L, A and c. Substituting (c, Q) and λ into (31), we know that $u_s > u_\tau$ iff $\frac{Q_s}{Q_\tau} > \frac{\varepsilon\gamma+1}{\gamma+1}$. Write the latter condition equivalently as follows:

$$\begin{aligned}&\tfrac{1}{\varepsilon-1}\ln\left[1-\delta(\varepsilon-1)\right]+\tfrac{\gamma}{\gamma+1}\ln\left[1-\delta\varepsilon(\gamma+1)\right]\\&-\tfrac{\varepsilon\gamma+1}{(\varepsilon-1)(\gamma+1)}\ln\left[1-\delta(\varepsilon-1)(\gamma+1)\right]+\ln(\gamma+1)-\tfrac{\gamma}{\gamma+1}\ln(\varepsilon\gamma+1)\quad>0.\end{aligned}$$

Temporarily denote the left-hand side as $LHS(\delta)$. It is clear that $LHS(\frac{1}{\varepsilon(\gamma+1)}) = -\infty$. Also, $LHS(0) > 0$ iff $\varepsilon < \varepsilon_0$, where ε_0 is defined in the proposition. Moreover, we can verify that

$$LHS'(\delta) = \frac{-\delta\gamma(\varepsilon\gamma+1)}{\left[1-\delta(\varepsilon-1)\right]\left[1-\delta(\varepsilon-1)(\gamma+1)\right]\left[1-\delta\varepsilon(\gamma+1)\right]} < 0.$$

If $\varepsilon \geq \varepsilon_0$, then $LHS(\delta) < LHS(0) \leq 0$, in which case $u_s < u_\tau$ for all $\delta \in (0, \bar{\delta})$. If $\varepsilon < \varepsilon_0$, then $LHS(0) > 0$, in which case there exists $\delta_0 \in (0, \bar{\delta})$ such that $u_s > u_\tau$ iff $\delta < \delta_0$.

For part (iii), recall that $u_s > u_\tau$ iff $\frac{Q_s}{Q_\tau} > \frac{\varepsilon\gamma+1}{\gamma+1}$. Because $\varepsilon > 1$, then $u_s > u_\tau$ implies $Q_s > Q_\tau$. Similarly, since $L = Q/\phi$ and $\phi_\tau > \phi_s$, then $Q_s > Q_\tau$ implies $L_s > L_\tau$. To compare the aggregate level of abatement and consumption of the clean good under the two polices, recall that $A = L\left[\left(\frac{bL}{M}\right)^\gamma - 1\right]$ and $c = w - wL\left(\frac{bL}{M}\right)^\gamma$ under the tax, while $A = L\left[\left(\frac{bL}{M}\right)^\gamma \lambda - 1\right]$ and $c = w - wL\left(\frac{bL}{M}\right)^\gamma \lambda$ under the standard. Since $\lambda > 1$, the inequality $L_s > L_\tau$ is sufficient for $A_s > A_\tau$ and $c_s < c_\tau$.

For part (iv), we can substitute equilibrium values of (c, Q) into the utility function to verify that $u(c + v(M)Q)$ depends on M entirely through the term $M[v(M)]^{(1+\gamma)/\gamma}$ and is increasing in this term. Then, part (iv) is evident. QED

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